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Modelling Fiscal Policy with OLGA

Treasury's OverLapping Generations model of the Australian economy

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Abstract

To meet the needs of a broad set of internal and external stakeholders now and into the future the Australian Treasury maintains a significant macroeconomic modelling capability. Treasury's current in-house capability is similar to that of the US Congressional Budget Office (CBO) and the Joint Committee on Taxation (JCT). These agencies have a suite of macroeconomic models to quantify the general equilibrium effects of policy on economic activity, household welfare and public finances. In this paper we introduce one of the models in that suite, Treasury's overlapping generations model of the Australian economy (hereafter OLGA). OLGA has been developed by economists in Treasury's Macroeconomic Analysis and Policy Division to support Treasury's counterfactual fiscal policy analysis. OLGA is a small open economy variant of the well-known lifecycle model developed by Auerbach and Kotlikoff (1987). It has been calibrated to Australian data and policy settings.

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Introduction

This paper introduces Treasury's overlapping generations model of the Australian economy (hereafter OLGA). OLGA has been developed in-house by economists in Treasury's Macroeconomic Analysis and Policy Division to support Treasury's counterfactual fiscal policy analysis. OLGA is a small open economy variant of the well-known lifecycle model developed by Auerbach and Kotlikoff (1987). It has been calibrated to Australian data and policy settings.

While no single economic framework can capture all the relevant dimensions of fiscal policy, overlapping generations (OLG) models have become one of the key tools in the economics profession for quantifying the effects of fiscal policy on macroeconomic aggregates, household welfare and public finances.[3] This reflects a number of desirable features of modern OLG models recognised by leading academics and fiscal agencies including the US Congressional Budget Office (CBO) and the Joint Committee on Taxation (JCT):

- OLG models are general equilibrium, meaning that they take full account of second round (and subsequent) effects in addition to the direct effects of policy changes. This includes taking account of the feedback effects of price changes and the spillovers from one market to another market.
- Households and firms are rational, forward-looking and maximise explicit objective functions subject to their budget constraints. This means agents' expectations are consistent with the economic framework and the modelled outcomes which permits an explicit comparison of household welfare across alternative fiscal policies.
- While the economy exists in perpetuity, households have a finite lifespan with the population comprising many overlapping generations of different ages at any point in time. This permits the inter-generational effect of fiscal policies to be identified.
- The inclusion of households of different skill levels, as well as different ages, introduces household heterogeneity. This allows an assessment of the effect of different policies on the distribution of household income and wealth, and on household welfare. OLG models are therefore able to quantify the effect of fiscal policy on cross-section and lifetime inequality.
- OLG models include a separate fiscal authority (hereafter government) that has a full set of taxation instruments at its disposal, undertakes expenditure and transfers, and is subject to a budget constraint that is binding in the long run. This means that the government can debt-finance a budget deficit in the short run but necessarily rules out policies that would result in unsustainable public finances in the long run. Changes in policy that have permanent fiscal effects must be offset by another policy change to ensure long-run budget neutrality. In contrast to government, households have a finite horizon. An important implication of these different decision horizons is

3 In the US context, for example, OLG models have been developed and used to investigate the economic efficiency of privatising social security (Nishiyama and Smetters (2007)), the intergenerational effect of closing the fiscal gap (Nishiyama and Reichling (2015)), the macroeconomic effects of reforming the Internal Revenue Code (JCT, 2018), and the budgetary and economic effects of a wealth tax (He et al. (2019)). In the Australian context, for example, OLG models have been developed and used to estimate the aggregate and distributional effects of various policy proposals to reform the retirement income system (Kudrna and Woodland (2010)), the behavioural effects of means test on pensioners (Tran and Woodland (2011)), the fiscal effects of demographic change (Kudrna et al. (2015)), and the economic and welfare consequences of different budget repair measures (Kudrna and Tran (2018)).

that strict Ricardian equivalence does not hold which means the model can identify fiscal policy that improves household welfare.

These features set OLG models apart from other macroeconomic modelling options available in Australia such as:

- Time series econometric models, macro-econometric models and other 'reduced form' approaches (for example, so called recursive CGE models) which lack a well-defined welfare criterion and sufficient detail to quantify the activity and welfare effects of alternative fiscal policies.
- Static general equilibrium models, including those previously used by Treasury and other researchers to study effects of different taxes in Australia (see for example, Cao et al. (2015), Murphy (2016) and KPMG (2016)), which ignore potentially important transitional dynamics and costs associated with adjusting aggregate capital or relocating capital across sectors.
- Representative agent dynamic general equilibrium models which are unable to consider intra- and inter-generational dimensions of fiscal policy because they assume a single infinitely-lived household.

Presented in this paper is OLGA Version 1.0 which has several key elements worth noting. First, the model has 75 generations of forward-looking, finite-lived households with five different skill types, ranging from high to low. This captures rich heterogeneity amongst Australian households. Second, the model has a number of production sectors based on the input-output table. This captures critical supply and demand relationships between producers and consumers within the domestic economy, and Australia's trade and financial linkages with the rest of the world (ROW). Third, the model has a detailed representation of major Australian taxes, age pension and other transfers and aggregate government spending. This allows Treasury to model Australian policy at the level of detail required for decision making. Finally, as a general equilibrium model including the indirect effects of a policy proposal, OLGA provides a comprehensive assessment of the so called 'dynamic cost' (or second round effects on the cost) of a policy proposal.

OLGA is a deterministic model which means it does not account for business cycle fluctuations and so is not well suited to modelling stabilisation policy. As such, it is best suited to analysing the allocative and distributional effects of fiscal policy.

Treasury follows best practice employed by other fiscal agencies (for example, CBO or JCT) by relying on a suite of economy-wide models to produce forecasts and policy advice. In addition to OLGA, Treasury's suite includes:

- Treasury's Macroeconometric Model of Australia (EMMA) (see Bullen, et al. (2021)). EMMA is central to Treasury's forecasting and policy advising process. EMMA is a large-scale macroeconometric model of the Australian economy. It captures the rich interaction between aggregate demand (which includes consumer spending, business investment, residential investment, government spending, and net exports) and aggregate supply (which includes the endogenous and exogenous factors that determine Treasury's estimate of potential output) in the economy. The interaction of aggregate demand and aggregate supply determines the forecasts of the other variables in the model, including inflation, interest rates, the unemployment rate and incomes. EMMA models policy using effective rates (for example, the average person income tax rate) or lump sum measures (for example, aggregate transfers to households).
- Treasury Industry Model (TIM) (see Carlton et al. (2023)). TIM has been developed to be Treasury's principal tool for industry policy analysis. It is a dynamic multisector general equilibrium model of the Australian economy. At its core TIM is a small open economy version of the neoclassical growth

model known as the Ramsey Cass Koopmans model. In contrast to typical neoclassical growth models, TIM has considerable firm heterogeneity in the form of 114 forward-looking, infinitely-lived firms that represent Australian industries. TIM includes significant detail on the linkages between industries and is well suited to studying the effects of industry-specific policies.

- Detailed analysis of the economic effects of policy proposals is generally conducted using OLGA, TIM or detailed bottom up analysis informed by empirical estimates from the literature. For materially large policies the effects are added to the EMMA baseline directly or by adjusting effective policy variables to yield a policy-consistent forecast of economic activity.

The development of OLGA, and Treasury's macroeconomic model development program more generally, has been supported by academic advisers with deep expertise in general equilibrium modelling, including OLG models.

The remainder of the paper is organised as follows: Section 2 describes in detail the theoretical structure of OLGA; Section 3 describes the techniques used to solve OLGA; Section 4 details the calibration methodology, including data sources; Section 5 describes the welfare criterion; and the paper concludes in Section 6. Other technical detail including OLGA's first order conditions can be found in Appendices A to G.

The model

Overview

OLGA is a variant of the well-known lifecycle model with overlapping generations developed by Auerbach and Kotlikoff (1987). The primary difference is that OLGA is a small open economy model calibrated to Australian data. The model includes households of multiple generations that are composed of workers with different skill levels, and a number of production sectors with different supply and demand side characteristics. A schematic diagram of OLGA is provided in Appendix A.

There are 75 overlapping generations of households in OLGA. Each generation enters the model at age 21 with a known earning ability, which reflects their age and level of skill. Households are assumed to be uniformly distributed across five skill types. Households save during their working lives and rely on savings during their retirement. Households' savings are managed by a notional funds manager. High net worth households leave intended bequests. Some households are eligible for means-tested retirement benefits at age 66. Households do not live past age 95.

In contrast to most OLG models, OLGA has seven production sectors, where each sector is characterised by a single representative firm that maximises the market value of the firm on behalf of its shareholders. Firms employ labour, capital, and intermediate inputs to produce final and intermediate goods and services for domestic use or export. Domestically produced goods compete with differentiated goods and services supplied by foreign producers. Firms manage capital investment decisions, and source funds from households and foreign investors via a notional funds manager.

OLGA incorporates a detailed representation of Australia's fiscal policies. These include major taxes such as progressive personal income tax with a range of tax offsets, corporate tax, goods and services tax, and major government transfers such as the means-tested age pension and other age-dependent payments. The government also has a fixed bundle of spending. The tax revenue finances government transfers and spending. The government relies on funds sourced from domestic and foreign investors to meet temporary primary deficits.

Households

Demographic structure

Population growth

The model is populated by households spanning 75 generations. Following the broader literature, a new household enters the model at age 21 and lives to a maximum age of 95. The age of a household is denoted by a , while the number of households of age a at the beginning of year t is denoted by H_t^a .

Given the model's time interval of one year, the total population at the beginning of year t is H_t which is the sum of the model's 75 generations:

$$H_t = \sum_{a=21}^{95} H_t^a$$

Population growth is exogenous to the model with the size of an incoming generation assumed to grow over time as follows:

$$H_{t+1}^{21} = H_t^{21}(1+\gamma_t^{h,21})$$

where $\gamma_t^{h,21}$ is the growth rate at time t of the population of 21-year-old households.

Households of age a survive until next period with a conditional probability of ψ_t^a and a cumulative probability $\Psi_t^a = \prod_{j=21}^{a} \psi_{t-j}^{j-1}$ of surviving until age a . This implies that the size of a generation declines over time. For example, the size of a generation that enters the model at the beginning of year s will be H_{s+a-21}^a when the cohort reaches age a :

$$H_{s+a-21}^a = H_{s+(a-1)-21}^{a-1}\psi_s^{a-1} = H_s^{21}\Psi_s^{a-1}$$
$$H_{s+a-21}^a \leq H_s^{21}$$

The implied growth of the total population is given by:

$$(1+\gamma_t^h) = H_{t+1} / H_t$$

where γ_t^h is the growth rate at time t of the total population.

Skill and technical progress

A worker's earning ability is an exogenous function of their age, skill level and the level of labour augmenting technical progress. Each generation is assumed to consist of households with five different skill levels denoted by ℓ . The size of the population of age a and skill ℓ at time t is denoted by $H_t^{a,\ell}$, with each skill type accounting for one-fifth of the generation (that is, $H_t^{a,\ell} = H_t^a / 5$). A worker's earning ability $\xi_t^{a,\ell}$ is assumed to be the product of their relative labour efficiency $\varsigma^{a,\ell}$ and labour augmenting technical progress ξ_t :

$$\xi_t^{a,\ell} = \varsigma^{a,\ell} \xi_t$$

Relative labour efficiency is a time-invariant function. To match lifetime participation and earnings profiles relative labour efficiency is assumed to be hump shaped, reflecting the accumulation of human capital over the course of the household's early life and following peak ability around age 40, a decline in ability that eventually settles at zero.

Labour-augmenting technical progress ξ_t is an exogenous process (determined outside the model) which is assumed to grow at rate γ_t^{ξ} , such that:

$$\xi_{t+1} = (1+\gamma_t^{\xi})\xi_t$$

Following the literature in describing the steady state of the model we normalise growing variables by their underlying exogenous trend. Normalised variables are denoted by a lower case. First, non-labour household variables and real wages are normalised by the level of labour augmenting technical progress. For example, the normalised consumption of a household of age a and skill ℓ at time t is $C_t^{a,\ell} / \xi_t = c_t^{a,\ell}$. Second, aggregate labour variables are normalised by aggregate population. For example, the normalised population of age a and skill ℓ at time t is $H_t^{a,\ell} / H_t = h_t^{a,\ell}$. Finally, aggregate non-labour variables are normalised by the product of labour augmenting technical progress and the aggregate population, which has a growth rate equal to $\gamma_t = (1+\gamma_t^h)(1+\gamma_t^{\xi})-1$. For example, the normalised aggregate exports at time t is $X_t / \left(\xi_t H_t\right) = x_t$.

Preferences

Households are assumed to maximise their lifetime utility. A representative household of skill type ℓ , who enters the model at the beginning of year s , is assumed to have the following lifetime utility function:

$$\sum_{a=21}^{95} \beta^{a-21} \left[\Psi_{s+a-21}^{a} U(c_{s+a-21}^{a,\ell}, L_{s+a-21}^{a,\ell}) + (1-\Psi_{s+a-21}^{a}) \Phi(v_{s+a-21}^{a,\ell}) \right] \qquad (1)$$

where: $0 < \beta < 1$ is the household's discount factor; $c_{s+a-21}^{a,\ell} \geq 0$, $0 \leq L_{s+a-21}^{a,\ell} \leq 1$, and $v_{s+a-21}^{a,\ell}$ are the household's consumption, leisure, and beginning-of-period savings at year $s+a-21$.

Given that there is a simple linear relationship between the year a household enters the model, their age and time, going forward the paper will denote household variables using only a time subscript t and an age superscript a .

Instantaneous utility

Instantaneous utility is assumed to be a constant relative risk aversion (CRRA) function of the household's aggregate consumption and leisure:

$$U(c_t^{a,\ell}, L_t^{a,\ell}) = \frac{\left[(c_t^{a,\ell})^{\alpha^\ell}(L_t^{a,\ell})^{(1-\alpha^\ell)}\right]^{(1-\sigma)}}{1-\sigma} \tag{2}$$

Where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and $0 < \alpha^\ell < 1$ measures the household's preference for consumption over leisure.

Consumption is a composite good sourced from a notional distributional sector which is explained in further details below.

Bequests

Following De Nardi (2004), we assume 'warm glow' altruism with the household deriving utility from bequeathing their residual savings following their death at time t and age a accordingly:

$$\Phi(v_{t+1}^{a+1,\ell}) = \frac{\phi_1^\ell(\phi_2^\ell + v_{t+1}^{a+1,\ell})^{1-\sigma}}{1-\sigma}$$

The term $\phi_1^\ell \geq 0$ measures the strength of the household's bequest motive, while $\phi_2^\ell > 0$ measures the extent to which bequests are a luxury good. A bequest motive will result in the build-up of savings that is more than the household would otherwise accumulate over its lifetime. For $\phi_2^\ell > 0$ the marginal utility of small bequests is bounded which suggests bequests are a luxury good.

For simplicity, we assume that savings of households who die at time t are distributed equally to their heirs who are assumed to be the surviving population of the same skill type:

$$vbq_{t+1}^{a+1,\ell} = \frac{\sum_{a=21}^{95}(1-\psi_t^a)H_t^{a,\ell}v_{t+1}^{a+1,\ell}}{\sum_{a=21}^{95}H_{t+1}^{a+1,\ell}} \tag{3}$$

Labour supply

For ease of exposition and without loss of generality, households are assumed to have one unit of time available for market-based work or leisure:

$$N_t^{a,\ell} = 1 - L_t^{a,\ell} \tag{4}$$

Aggregate labour supply measured in units of time is given by:

$$N_t = \sum_a \sum_\ell N_t^{a,\ell} H_t^{a,\ell}, \tag{5}$$

while aggregate labour supply measured in relative efficiency units is given by:

$$\tilde{N}_t = \sum_a \sum_\ell \xi^{a,\ell} N_t^{a,\ell} H_t^{a,\ell} \qquad (6)$$

Efficiency units are perfectly substitutable. As such, workers earn a common before tax wage of w_t per efficiency unit. The hourly wage received by a household of age a and skill ℓ, $w_t^{a,\ell}$ is the product of the common wage rate per efficiency unit w_t and the household's relative labour efficiency:

$$w_t^{a,\ell} = w_t \xi^{a,\ell}$$

Savings

Households have a single savings instrument which is assumed to be managed by a notional financial manager. The value of household savings at the beginning of time t is denoted by $v_t^{a,\ell}$. The underlying asset is a portfolio of domestic and internationally traded assets. The before personal income tax rate of return for this asset is r_t^h. Details of the funds market are discussed in a later section.

Households enter the model with zero savings net of bequests (that is, $v_t^{21,\ell} - vbq_t^{21,\ell} = 0$). However, they can borrow at the before personal income tax rate of return r_t^h subject to the following borrowing constraint:

$$v_t^{a,\ell} \geq \underline{v}_t^{a,\ell}$$

where $\underline{v}_t^{a,\ell} \leq 0$ is the credit limit for households to borrow. For the current version of OLGA, we have $\underline{v}_t^{a,\ell} = -\infty$ for all $a \leq 60$ and $\underline{v}_t^{a,\ell} = 0$ for all $a > 60$. Intuitively, this allows households under the age of 60 (but not over) to borrow against their future income. This means that households who are eligible for age pensions are not able to borrow against future age pension payments.

Personal income tax

Households are subject to personal income tax (PIT). PIT is levied on the following tax base:

$$yh_t^{a,\ell} = \left(1 - dt_t^{a,\ell}\right)\left(yn_t^{a,\ell} + yv_t^{a,\ell} + pen_t^{a,\ell} + ben_t^{a,\ell}\right)$$

where: $dt_t^{a,\ell}$ is the ratio of income tax deductions and exemptions to gross income; $pen_t^{a,\ell}$ is the means-tested age pension; $ben_t^{a,\ell}$ is other taxable transfers from the government such as unemployment benefits and the disability pension; and taxable labour income before deductions $yn_t^{a,\ell}$ is defined as follows:

$$yn_t^{a,\ell} = w_t^{a,\ell} N_t^{a,\ell}$$

The capital income tax base for the purpose of calculating personal income tax payable $yv_t^{a,\ell}$ is:

$$yv_t^{a,\ell} = \tilde{r}_t^h v_t^{a,\ell} + dv_t^{a,\ell}$$

where: $\tilde{r}_t^h$ is the taxable return on savings at time t, which reflects the actual distribution/flow of earnings via interest, dividends and off-market share buy-backs; and $dv_t^{a,\ell}$ denotes changes to the value of savings due to changes in asset prices. Taxable returns are also affected by discounting and credits.

Personal income tax payable (before franking credit refunds and any offsets) varies according to the following progressive schedule with *J* brackets:

$$gpit_t^{a,\ell} = \begin{cases} \tau_{0,t}^{pit}(yh_t^{a,\ell} - \overline{yh}_{0,t}) & \text{if } yh_t^{a,\ell} \le \overline{yh}_{1,t} \\ \sum_{j=2}^{J*} \tau_{j-2,t}^{pit}(\overline{yh}_{j-1,t} - \overline{yh}_{j-2,t}) + \tau_{J*-1,t}^{pit}(yh_t^{a,\ell} - \overline{yh}_{J*-1,t}) & \text{if } \overline{yh}_{1,t} < yh_t^{a,\ell} \le \overline{yh}_{J*,t} \\ \sum_{j=2}^{J+1} \tau_{j-2,t}^{pit}(\overline{yh}_{j-1,t} - \overline{yh}_{j-2,t}) + \tau_{J,t}^{pit}(yh_t^{a,\ell} - \overline{yh}_{J,t}) & \text{if } yh_t^{a,\ell} > \overline{yh}_{J,t} \end{cases}$$

where: $J*$ is the closest threshold that is larger than $yh_t^{a,\ell}$; $\overline{yh}_{j,t}$ is the upper threshold for income bracket j; and income in bracket $\overline{yh}_{j-1,t}$ to $\overline{yh}_{j,t}$ is subject to marginal tax rate $\tau_{j,t}^{pit}$.

The gross personal income tax liability can be reduced by a number of income-dependent tax offsets:

$$offset_{j,t}^{a,\ell} = \begin{cases} offsetmax_{j,t} & \text{for all } yh_t^{a,\ell} \le \overline{yh}_{1,j,t}^{o} \\ offsetmax_{j,t} - \omega_{j,t}^{o}(yh_t^{a,\ell} - \overline{yh}_{1,j,t}^{o}) & \text{for all } \overline{yh}_{1,j,t}^{a,\ell} < yh_t^{a,\ell} < \overline{yh}_{2,j,t}^{o} \\ 0 & \text{for all } yh_t^{a,\ell} \ge \overline{yh}_{2,j,t}^{o} \end{cases}$$

where: $offset_{j,t}^{a,\ell}$ is the maximum amount for offset j; $\overline{yh}_{1,j,t}^{o}$ and $\overline{yh}_{2,j,t}^{o}$ are income-test thresholds; and $\omega_{j,t}^{o}$ is the taper rate, which indicates the amount the offset is reduced by for each additional unit of income above $\overline{yh}_{1,j,t}^{o}$.

The net personal income tax liability is equal to the gross liability less franking credits and tax offsets. Tax offsets are non-refundable and are therefore applied before franking credits, resulting in a net personal income tax liability as follows:

$$pit_t^{a,\ell} = \max\left\{0, gpit_t^{a,\ell} - \sum_{j=1}^{J} offset_{j,t}^{a,\ell}\right\} - yf_t^{a,\ell}$$

where: $yf_t^{a,\ell}$ is the franking credit received by the household.

Given the personal income tax system described above, the effective marginal personal income tax rate will be a combination of a given household's marginal tax rate $\tau_{j,t}^{pit}$, and the taper rates $\omega_{j,t}^{o}$ for the tax offsets that the household receives.

Transfers

Age pension

A household may be entitled to age pension $pen_t^{a,\ell}$ after reaching the pension eligibility age, subject to asset and income means testing. Households receive the smaller of the pensions implied by the asset and income tests:

$$pen_t^{a,\ell} = \begin{cases} 0 & \text{for all } a < pa_t \\ \min\left\{pen_t^{v}, pen_t^{y}\right\} & \text{for all } a \geq pa_t \end{cases}$$

where: pen_t^{v} is the asset-tested pension; pen_t^{y} is the income-tested pension; and pa_t is the pension eligibility age.

The asset-tested pension is determined according to the following rule:

$$pen_t^{v} = \begin{cases} pmax_t & \text{for all } \tilde{v}_t^{a,\ell} \leq \overline{v}_{1,t} \\ pmax_t - \omega_t^{v}(\tilde{v}_t^{a,\ell} - \overline{v}_{1,t}) & \text{for all } \overline{v}_{1,t} < \tilde{v}_t^{a,\ell} < \overline{v}_{2,t} \\ 0 & \text{for all } \tilde{v}_t^{a,\ell} \geq \overline{v}_{2,t} \end{cases}$$

where: $\tilde{v}_t^{a,\ell}$ are assessable assets ($\tilde{v}_t^{a,\ell} = v_t^{a,\ell} - vd_t^{a,\ell}$, where $vd_t^{a,\ell}$ are exempt assets); $\overline{v}_{1,t}$ and $\overline{v}_{2,t}$ are the asset-test thresholds; and ω_t^{v} is the taper rate, which indicates the amount the pension is reduced by for each additional unit of asset above $\overline{v}_{1,t}$.

The maximum pension payment $pmax_t$ is in turn given by:

$$pmax_t = \phi_t^{pen} w_t \overline{N}$$

Where $\overline{N}$ and ϕ_t^{pen} are the levels of household average labour supply and the replacement ratio consistent with the current pension rule. This mimics the reality that age pensions are indexed to current price levels and benchmark earnings.[4]

Similarly, the income-tested pension is determined according to the following rule:

$$pen_t^{y} = \begin{cases} pmax_t & \text{for all } yh_t^{*a,\ell} \leq \overline{yh_{1,t}} \\ pmax_t - \omega_t^{y}\left(yh_t^{*a,\ell} - \overline{yh_{1,t}}\right) & \text{for all } \overline{yh_{1,t}} < yh_t^{*a,\ell} < \overline{yh_{2,t}} \\ 0 & \text{for all } yh_t^{*a,\ell} \geq \overline{yh_{2,t}} \end{cases}$$

where $yh_t^{*a,\ell}$ is the sum of a household's labour income and deemed income from savings:

$$yh_t^{*a,\ell} = yn_t^{a,\ell} + yv_t^{*a,\ell}$$

4 For reference, see https://www.dss.gov.au/seniors/benefits-payments/age-pension.

where: $\overline{yh_{1,t}}$ and $\overline{yh_{2,t}}$ are income-test thresholds; and ω_t^y is the taper rate which indicates the amount the pension is reduced for each additional unit of income above $\overline{yh_{1,t}}$.

The deemed income from savings $yv_t^{*a,\ell}$ is derived using deeming rates $\omega_t^{dm2} > \omega_t^{dm1} > 0$ and a deeming threshold $v_t^{dm} > 0$:

$$yv_t^{*a,\ell} = \begin{cases} \omega_t^{dm1} v_t^{a,\ell} & \text{if } v_t^{a,\ell} \le v_t^{dm} \\ \omega_t^{dm1} v_t^{dm} + \omega_t^{dm2}(v_t^{a,\ell} - v_t^{dm}) & \text{if } v_t^{a,\ell} > v_t^{dm} \end{cases}$$

Other transfers

Households also receive lump-sum transfers (if positive) or pay lump-sum tax (if negative) denoted by $tr_t^{a,\ell}$. The lump-sum transfer is the sum of government benefits and transfers that are specific to certain households $ben_t^{a,\ell}$ and a general lump-sum transfer/tax that applies to all households ls_t.

Household budget constraint and optimisation problem

The household's flow budget constraint is as follows:

$$(1+\gamma_t^{\xi})(v_{t+1}^{a+1,\ell} - vbq_{t+1}^{a+1,\ell}) + p_t^c c_t^{a,\ell} = (1+r_t^h)v_t^{a,\ell} + w_t^{a,\ell} N_t^{a,\ell} - pit_t^{a,\ell} + pen_t^{a,\ell} + tr_t^{a,\ell} \qquad (7)$$

Here p_t^c is the after-tax price of household's aggregate consumption bundle at time t.

The household's objective is to maximise lifetime utility (1), subject to the budget constraint (7). Further details of the households' optimisation problem are provided in Appendix B.

Production sector

For simplicity, we assume that there are J sectors with a single firm in each representing a large number of individual firms with homogenous technology trading in a perfectly competitive market. In other words, there is one price-taking firm in each sector j.

Production technology

The firm's production technology is represented by a constant elasticity of substitution (CES) function:

$$F(\lambda_{j,t}, \tilde{N}_{j,t}, K_{j,t}, Z_{j,t}) = \lambda_{j,t}\left[\theta_j^{yn}(\xi_t \tilde{N}_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}} + \theta_j^{yk}(K_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}} + \theta_j^{yz}(Z_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}}\right]^{\frac{\eta_j^y}{\eta_j^y - 1}} \qquad (8)$$

where for sector j : $\lambda_{j,t}$ is total factor productivity; $\tilde{N}_{j,t}$ is the sector's use of labour input measured in relative efficiency units;[5] $K_{j,t}$ is the sector specific capital stock; $Z_{j,t}$ is a composite intermediate input; $0<\theta_j^{yn}<1$, $0<\theta_j^{yk}<1$, and $0<\theta_j^{yz}<1$ are the CES weights for each input, with $\theta_j^{yn}+\theta_j^{yk}+\theta_j^{yz}=1$; and η_j^y is the elasticity of substitution for factors of production.

The normalised version of equation (8) is:

$$\Gamma(\lambda_{j,t},\tilde{n}_{j,t},k_{j,t},z_{j,t})=\lambda_{j,t}\left[\theta_j^{yn}(\tilde{n}_{j,t})^{\frac{\eta_j^y-1}{\eta_j^y}}+\theta_j^{yk}(k_{j,t})^{\frac{\eta_j^y-1}{\eta_j^y}}+\theta_j^{yz}(z_{j,t})^{\frac{\eta_j^y-1}{\eta_j^y}}\right]^{\frac{\eta_j^y}{\eta_j^y-1}} \quad (9)$$

Labour input

Labour is supplied by households. Efficiency units of labour are perfectly substitutable across sectors, so in equilibrium workers, in different sectors, receive a single wage. As such firms and workers are indifferent to the sectoral allocation of workers by efficiency unit:

$$\tilde{n}_t=\sum_{j=1}^{J}\tilde{n}_{j,t} \quad (10)$$

Capital input

Firms own and manage their variable productive capital. The capital stock of a representative firm in sector j evolves according to the following accumulation identity:

$$(1+\gamma_t)k_{j,t+1}=(1-\delta_{j,t})k_{j,t}+i_{j,t} \quad (11)$$

Investment is a composite good sourced from a notional distribution sector which is explained in further details below.

Following Lucas (1967), we assume that the firm faces capital adjustment costs. In particular, we assume that the firm faces quadratic adjustment costs such that there is an increasing loss of output as the investment to capital ratio moves further away from its steady state level:

$$\Omega_j^k(i_{j,t},k_{j,t})=\frac{\zeta_j^k}{2}\left(\frac{i_{j,t}}{k_{j,t}}-\delta_{j,t}-\gamma_t\right)^2 k_{j,t} \quad (12)$$

where $\zeta_j^k \geq 0$ governs the size of the adjustment cost.

5 Because efficiency units are perfect substitutes, we cannot identify the share of total hours employed by the representative firm in sector *j*.

Intermediate inputs

Production requires the consumption of intermediate inputs in the period in which they are produced. For example, the production of bread requires intermediate inputs such as flour, yeast and salt.

While intermediate inputs are directly sourced by firms from each production sector, for ease of exposition we assume that firms purchase a sector-specific composite of intermediate goods from a notional distribution sector which is described in more detail below.

Gross output

Finally, the firm's gross output is summarized by the following:

$$y_{j,t} = \Gamma(\lambda_{j,t}, \tilde{n}_{j,t}, k_{j,t}, z_{j,t}) - \Omega_j^k(i_{j,t}, k_{j,t}) \quad (13)$$

Corporate finance

The representative firm owns the capital stock and makes production and investment decisions in order to maximise the market value of the firm. In addition to equity the firm's capital is also financed by borrowing in the corporate debt market. For simplicity we assume that firms maintain fixed debt to equity ratios. Furthermore, to capture features of the Australian tax system equity is divided into equity which pays dividends or capital gains.

The value of the firm's capital financed by corporate debt is:

$$b_{j,t}^c = \mu_{j,t}^b p_{j,t}^k k_{j,t}$$

where: $p_{j,t}^k$ is the market value of a unit of capital; $k_{j,t}$ is the capital stock at the beginning of period t; and $\mu_{j,t}^b$ is the share of the firm's capital financed by corporate debt. For Version 1.0 of OLGA, $\mu_{j,t}^b$ is exogenous.

Corporate debt is assumed to be an internationally traded one period bond which has a required rate of return of r_t^c, which means that each period, in addition to the principal, the firm must also pay $r_t^c b_{j,t}^c$ to bond holders.

The representative firm's equity (that is, the market value of the firm) is:

$$\begin{aligned} v_{j,t}^c &= p_{j,t}^k k_{j,t} - b_{j,t}^c \\ &= (1-\mu_{j,t}^b) p_{j,t}^k k_{j,t} \end{aligned} \quad (14)$$

Corporate equity is also traded internationally. We assume that the required rate of return for equity is set by the global funds market based on financial assets with the same risk characteristics.

We assume that a notional funds manager raises funds on behalf of firms. The funds manager sources funds from domestic households and foreign investors. Details of the funds market are discussed in a later section.

Market value of the firm

The no-arbitrage condition requires that:

$$\frac{Q_{j,t}+(V^{c}_{j,t+1}-V^{c}_{j,t})}{V^{c}_{j,t}}=r^{e}_{t} \tag{15}$$

where: $Q_{j,t}$ is the net cash-flow from the firm to shareholders; $V^{c}_{j,t}$ is the market value of the firm's equity at the beginning of period t; and r^{e}_{t} is the required rate of return for equity. This condition implies that investors are indifferent to holding equity (with earnings returned via dividends or capital gain) and holding other financial assets with the same risk characteristics. The net cash-flow $Q_{j,t}$ may reflect outflows due to dividends and/or off-market share buy-back and inflows from the issuance of new equity.

Using normalised variables, we have:

$$\frac{q_{j,t}+(1+\gamma_t)v^{c}_{j,t+1}-v^{c}_{j,t}}{v^{c}_{j,t}}=r^{e}_{t} \tag{16}$$

where: $q_{j,t}$ is the net cash-flow from firms to households; and $v^{c}_{j,t}$ is the market value of the firm's equity at the beginning of period t.

This implies:

$$v^{c}_{j,t}=\frac{q_{j,t}+(1+\gamma_t)v^{c}_{j,t+1}}{1+r^{e}_{t}} \tag{17}$$

Forward substitution of the no-arbitrage condition (assuming there are no self-fulfilling speculative asset-price bubbles) implies the market value of the firm's equity is equivalent to the present value of current and all future net cash-flow to shareholders:

$$v^{c}_{j,t}=\sum_{s=t}^{\infty}\left(\prod_{u=t}^{s}\frac{1+\gamma_u}{1+r^{e}_{u}}\right)\left(\frac{q_{j,s}}{1+\gamma_s}\right) \tag{18}$$

Corporate income tax

The firm's earnings before interest, tax and amortisation is:

$$e_{j,t}=p^{y}_{j,t}y_{j,t}-(1+\tau^{prt}_{j,t})w_t\tilde{n}_{j,t}-p^{z}_{j,t}z_{j,t} \tag{19}$$

where: $p^{y}_{j,t}$ is the producer's price; $y_{j,t}$ is real gross production; $\tau^{prt}_{j,t}$ is the payroll tax rate; $\tilde{n}_{j,t}$ is the sector's use of labour inputs measured in efficiency units; $z_{j,t}$ is a composite of intermediate inputs from all production sectors (including itself); and $p^{z}_{j,t}$ is the price of the composite intermediate good.

Corporate income tax (CIT) is payable to the government. Given deductions on interest payable on corporate debt, investment allowances and depreciation, a firm pays the following level of CIT:

$$cit_{j,t} = \tau^{cit}_{j,t}(e_{j,t} - r^c_t b^c_{j,t} - \phi^k_{j,t}\delta_{j,t} p^i_{j,t} k_{j,t} - \phi^i_{j,t} p^i_{j,t} i_{j,t}) \quad (20)$$

where: $\tau^{cit}_{j,t}$ is the effective CIT rate; $\phi^k_{j,t}$ is the proportion of economic depreciation that is tax deductible; $\delta_{j,t}$ is the economic depreciation rate; $\phi^i_{j,t}$ is an additional investment allowance that can be deducted as an expense under the tax system; $i_{j,t}$ is the level of investment and $p^i_{j,t}$ is market price of investment goods.

The effective CIT rate is the product of the statutory CIT rate τ^{cit}_t and the CIT taxable share of the sector $\phi^{cit}_{j,t}$:

$$\tau^{cit}_{j,t} = \tau^{cit}_t \phi^{cit}_{j,t}$$

where $\phi^{cit}_{j,t}$ reflects the share of firms (on an output value basis) that are incorporated and therefore subject to corporate tax. The current version of the model assumes $\phi^{cit}_{j,t}$ is exogenous which means firms cannot optimise their value subject to legal structure. The main implications of this assumption are that changes to personal income tax settings have no direct effect on the investment decisions in the unincorporated sector, and that depreciation of capital in the unincorporated sector cannot be deducted as an expense for tax purposes. Similarly, there is currently no provision for carrying forward losses in the model. These limitations will be addressed in future model developments.

Payroll tax

Firms are also subject to payroll tax, which is payable to the government:

$$prt_{j,t} = \tau^{prt}_{j,t} w_t \tilde{n}_{j,t} \quad (21)$$

Firm's budget constraint

The firm is subject to the following flow budget constraint:

$$(1 + r^c_t) b^c_{j,t} + cit_{j,t} + q_{j,t} + p^i_{j,t} i_{j,t} = e_{j,t} + (1 + \gamma_t) b^c_{j,t+1} \quad (22)$$

Earnings plus borrowing must equal the sum of corporate bond repayment (principal and interest), corporate tax, investment and net cash flow paid to shareholders (dividends or off-market share buy-backs less new share issuance).

Dividend policy

For simplicity, we assume the firm retains sufficient earnings to cover the cost of replacing depreciated capital $\delta_{j,t} p^i_{j,t} k_{j,t}$. After interest $r^c_{j,t} b^c_{j,t}$ and tax $cit_{j,t}$, the balance is returned as either a dividend or off-market share buy-back collectively denoted by $d_{j,t}$:

$$d_{j,t} = e_{j,t} - r^c_{j,t} b^c_{j,t} - cit_{j,t} - \delta_{j,t} p^i_{j,t} k_{j,t}$$

This implies the following rule for the new issuance of equity $nv^c_{j,t}$:

$$\begin{aligned} nv^c_{j,t} &= d_{j,t} - q_{j,t} \\ &= p^i_{j,t} i_{j,t} - \delta_{j,t} p^i_{j,t} k_{j,t} - (1+\gamma_t) b^c_{j,t+1} + b^c_{j,t} \end{aligned}$$

The new issuance of shares is equal to the cost of investment in excess of depreciation less the change in the value of the firm's capital that is financed by bonds. At the steady state, this rule implies new issuance is equal to the growth of the capital stock financed by equity:

$$\begin{aligned} nv^c_{j,\infty} &= p^i_{j,\infty} i_{j,\infty} - \delta_{j,\infty} p^i_{j,\infty} k_{j,\infty} - \gamma_\infty b^c_{j,\infty} \\ &= (\gamma_\infty + \delta_{j,\infty}) p^i_{j,\infty} k_{j,\infty} - \delta_{j,\infty} p^i_{j,\infty} k_{j,\infty} - \gamma_\infty \mu^c_{j,\infty} p^i_{j,\infty} k_{j,\infty} \\ &= \gamma_\infty (1 - \mu^b_{j,\infty}) p^i_{j,\infty} k_{j,\infty} \\ &= \gamma_\infty (1 - \mu^b_{j,\infty}) p^k_{j,\infty} k_{j,\infty} \end{aligned}$$

Firm's optimisation problem

The representative firm's objective is to maximise the market value of the firm (18) by choosing the level of production (that is, the level of capital, labour and intermediate inputs) subject to the budget constraint (22), production technology (13), and the market prices of output, investment, labour and intermediate goods. Further details of the firm's optimisation problem are provided in Appendix C.

Rest of the world

Global goods and services market

For imports of goods and services, we assume the foreign sector supplies $m_{j,t}$ at the exogenously determined price $p^m_{j,t}$. This implies that domestic demand cannot affect the price. In other words, the supply of imports is perfectly elastic at this price. Imports compete with differentiated goods of the same type produced domestically. The extent to which they compete is dictated by the degree of substitutability. For ease of exposition, we assume there is a notional distribution sector which aggregates domestic and imported varieties of goods. This approach is explained in detail in the next section.

For exports of goods and services, we adopt a richer specification than typically assumed in small open economy models by recognising that export demand is affected by substitutability for varieties of the same good; substitutability for other goods; and the market power of Australian producers. In a similar way to global economic models (for example, Hertel (1997); McKibbin and Wilcoxen (1998)), we employ a representative consumer in the ROW who solves a nested utility maximisation problem. At the top of the nest is the choice between different types of goods. Following this is the choice between varieties of the same good that are produced in different exporting countries.

As derived in Appendix D, the foreign consumer's optimal level of demand for Australian exports are:

$$x_{j,t} = \theta_j^{cm*} \left(\frac{p_{j,t}^{y}}{p_{j,t}^{c*}} \right)^{-\eta_j^{cm*}} \theta_j^{c*} \left(\frac{p_{j,t}^{c*}}{p_t^{c*}} \right)^{-\eta^{c*}} c_t^* \tag{23}$$

where for good j : $x_{j,t}$ is the level of Australian exports; c_t^* is the level of foreign aggregate demand; $0 < \theta_j^{c*} < 1$ is the CES weight for the specific good in foreign aggregate demand; $0 < \theta_j^{cm*} < 1$ is the CES weight for Australian exports in foreign demand for the specific good; $\eta^{c*} > 0$ is the elasticity of substitution between different types of goods; and $\eta_j^{cm*} > 0$ is the elasticity of substitution between varieties of the same good from different exporters. Furthermore, p_t^{c*} is the price index of the foreign consumer's aggregate demand, and $p_{j,t}^{c*}$ is the price index of good j for the foreign consumer that is a CES aggregate of the prices of exports the from Australia $p_{j,t}^{y}$ and the ROW $p_{j,t}^{y*}$:

$$p_{j,t}^{c*} = \left(\theta_j^{cm*} \left(p_{j,t}^{y} \right)^{1-\eta_j^{cm*}} + \left(1 - \theta_j^{cm*}\right) \left(p_{j,t}^{y*} \right)^{1-\eta_j^{cm*}} \right)^{\frac{1}{1-\eta_j^{cm*}}} \tag{24}$$

Both p_t^{c*} and $p_{j,t}^{y*}$ are assumed to be exogenous.

The export demand function for good j makes it clear that Australian exports depend on the substitutability of Australian and ROW varieties of good j , η_j^{cm*} , the substitutability of all goods η^{c*} and the market power of Australian producers of good j , θ_j^{cm*} . When competing with other varieties, higher substitutability implies export demand is more sensitive to price changes, while greater market power implies export demand is less sensitive to price changes. When competing with other goods, higher substitutability η_j^{c*} implies export demand is more sensitive to aggregate price changes, while greater market power θ_j^{cm*} implies greater influence of Australian prices on aggregate prices.

Global funds market

Foreign capital is assumed to be supplied as one period bonds or equity. Irrespective of the end use, the supply of capital is assumed to be perfectly elastic at the global required after tax rate of return for that investment type. Required rates of return vary across investment types due to different risk premia. We assume a benchmark rate equal to the global required after tax rate of return for government bonds denoted r_t^{g*} . Following the empirical literature, equity is assumed to have a larger risk premium than corporate debt, which in turn has a larger premium than sovereign debt.

Finally, Australian investors can also undertake portfolio investment offshore at an assumed foreign after-tax rate of return r_t^{f*} .

National budget constraint

Given the quantities and prices of imports and exports, as well as the global required rates of return, the national budget constraint is as follows:

$$(1+\gamma_t)(v_{t+1}^{f}-v_{t+1}^{f*})=\sum_{j}^{J}p_{j,t}^{y}x_{j,t}-\sum_{j}^{J}p_{j,t}^{m}m_{j,t}+(1+r_t^{f*})v_t^{f}-(1+r_t^{f})v_t^{f*} \quad (25)$$

where: v_t^f are household assets held abroad; v_t^{f*} are gross assets held in Australia by foreigners; and r_t^f is the average after-tax rate of return for the foreigner's investment portfolio.

Distribution sector

As discussed above, for ease of exposition we assume that there is a notional domestic distribution sector. The distribution firm combines imported goods and domestically produced goods to form composite consumption $c_t^{a,\ell}$, investment $i_{j,t}$, intermediate inputs $z_{j,t}$, and government spending g_t which are sold to households, firms and the government. Further details of the distribution sector's optimisation problem are provided in Appendix E.

Consumption goods and services

Household consumption $c_t^{a,\ell}$ is a nested CES aggregate of individual consumption goods which reflects household preferences. At the top tier, the distribution sector notionally allocates total expenditure $p_t^c c_t^{a,\ell}$ to a basket of goods $c_{j,t}^{a,\ell}$, which are in turn composites of domestic and imported varieties. The distribution sector picks the consumption bundle that maximises household utility subject to the allocated budget:

$$\max_{\{c_{j,t}^{a,\ell}\}} c_t^{a,\ell}=\left(\sum_j \theta_j^c\left(c_{j,t}^{a,\ell}\right)^{\frac{\eta^c-1}{\eta^c}}\right)^{\frac{\eta^c}{\eta^c-1}}$$

subject to:

$$p_t^c c_t^{a,\ell}\geq\sum_j p_{j,t}^c c_{j,t}^{a,\ell}$$

where: $\eta^c>0$ is the elasticity of substitution; $0<\theta_j^c<1$ is the CES weight for different goods j with $\sum_j\theta_j^c=1$; and $p_{j,t}^c$ is the market price including all taxes and subsidies on each composite good.

At the bottom tier, the distribution sector notionally allocates expenditure $p_{j,t}^c c_{j,t}^{a,\ell}$ to a basket of goods composed of domestically produced $cd_{j,t}^{a,\ell}$ and imported $cm_{j,t}^{a,\ell}$ varieties. The distribution sector picks the consumption bundle that maximises household utility subject to the allocated budget:

$$\max_{cd_{j,t}^{a,\ell},cm_{j,t}^{a,\ell}} c_{j,t}^{a,\ell}=\left(\left(1-\theta_j^{cm}\right)\left(cd_{j,t}^{a,\ell}\right)^{\frac{\eta_j^{cm}-1}{\eta_j^{cm}}}+\left(\theta_j^{cm}\right)\left(cm_{j,t}^{a,\ell}\right)^{\frac{\eta_j^{cm}-1}{\eta_j^{cm}}}\right)^{\frac{\eta_j^{cm}}{\eta_j^{cm}-1}}$$

subject to:

$$p_{j,t}^{c} c_{j,t}^{a,\ell} \geq (1+\omega_{j,t}^{c}\tau_t^{gst})(1+\tau_{j,t}^{oit,c}) p_{j,t}^{y} cd_{j,t}^{a,\ell} + (1+\omega_{j,t}^{c}\tau_t^{gst})(1+\tau_{j,t}^{oit,c}) p_{j,t}^{m} cm_{j,t}^{a,\ell}$$

where: η_j^{cm} is the elasticity of substitution; $0 < \theta_j^{cm} < 1$ is the CES weight for imported goods; τ_t^{gst} is the statutory goods and services tax (GST); $\omega_{j,t}^{c}$ is the share of goods subject to GST; and $\tau_{j,t}^{oit,c}$ is an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of consumption goods and services.

Investment goods and services

Capital formation of sector j is assumed to be a composite investment good $i_{j,t}$ which is an aggregate of different types of goods $i_{j,k,t}$ which in turn is a composite of domestically produced $id_{j,k,t}$ and imported $im_{j,k,t}$ varieties.

At the top-tier, the distribution sector's objective is to maximise the value of the firm's gross fixed capital expenditure by combining different types of investment goods subject to the firm's investment preferences, captured by a CES function, and the firm's allocated budget $p_{j,t}^{i} i_{j,t}$:

$$\max_{\{i_{j,k,t}\}} i_{j,t} = \left(\sum_k \theta_{j,k}^{i} \left(i_{j,k,t} \right)^{\frac{\eta_j^i - 1}{\eta_j^i}} \right)^{\frac{\eta_j^i}{\eta_j^i - 1}}$$

subject to the budget constraint:

$$p_{j,t}^{i} i_{j,t} \geq \sum_k p_{j,k,t}^{i} i_{j,k,t}$$

where: $\eta_j^{i} > 0$ is the elasticity of substitution; $0 < \theta_{j,k}^{i} < 1$ is the CES weight for variety k with $\sum_k \theta_{j,k}^{i} = 1$; and $p_{j,k,t}^{i}$ is the market price of each composite good for investment of the sector.

At the bottom-tier, the distribution sector's objective is to maximise the firm's value of gross fixed capital expenditure of variety k by combining domestic and imported varieties of investment goods subject to the firm's investment preferences, captured by a CES function, and the firm's allocated budget $p_{j,k,t}^{i} i_{j,k,t}$:

$$\max_{id_{j,k,t}, im_{j,k,t}} i_{j,k,t} = \left(\left(1-\theta_{j,k}^{im}\right)\left(id_{j,k,t}\right)^{\frac{\eta_{j,k}^{im}-1}{\eta_{j,k}^{im}}} + \left(\theta_{j,k}^{im}\right)\left(im_{j,k,t}\right)^{\frac{\eta_{j,k}^{im}-1}{\eta_{j,k}^{im}}} \right)^{\frac{\eta_{j,k}^{im}}{\eta_{j,k}^{im}-1}}$$

subject to:

$$p_{j,k,t}^{i} i_{j,k,t} \geq (1+\omega_{j,k,t}^{i}\tau_t^{gst})(1+\tau_{j,k,t}^{oit,i}) p_{j,t}^{y} id_{j,k,t} + (1+\omega_{j,k,t}^{i}\tau_t^{gst})(1+\tau_{j,k,t}^{oit,i}) p_{j,t}^{m} im_{j,k,t}$$

where: $\eta_{j,k}^{im}$ is the elasticity of substitution; $0<\theta_j^{im}<1$ is the CES weight for imported goods; $\omega_{j,k,t}^{i}$ is share of investment goods subject to the GST; and $\tau_{j,k,t}^{oit,i}$ is an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of investment goods and services.

Intermediate goods and services

In a similar vein, the distribution sector produces aggregate intermediate good $z_{j,t}$ by combining different inputs $z_{j,k,t}$, which are in turn composites of domestically produced $zd_{j,k,t}$ and imported $zm_{j,k,t}$ varieties.

At the top-tier, the distribution sector's objective is to maximise the value of the firm's intermediate input expenditure by combining different types of intermediate goods subject to the firm's production requirements, captured by a CES function, and the firm's allocated budget $p_{j,t}^{z}z_{j,t}$:

$$\max_{\{z_{j,k,t}\}} z_{j,t}=\left(\sum_k \theta_{j,k}^{z}\left(z_{j,k,t}\right)^{\frac{\eta_j^z-1}{\eta_j^z}}\right)^{\frac{\eta_j^z}{\eta_j^z-1}}$$

subject to the expenditure constraint:

$$p_{j,t}^{z}z_{j,t}\geq\sum_k p_{j,k,t}^{z}z_{j,k,t}$$

where: $\eta_j^z>0$ is the elasticity of substitution; $0<\theta_{j,k}^{z}<1$ is the CES weight for variety k with $\sum_k \theta_{j,k}^{z}=1$; and $p_{j,k,t}^{z}$ is the market price of intermediate good k.

At the bottom-tier, the distribution sector's objective is to maximise the firms value of intermediate goods expenditure on variety k by combining domestic and imported varieties of intermediate goods subject to the firm's production preferences, captured by a CES function, and the firm's allocated budget $p_{j,k,t}^{z}z_{j,k,t}$:

$$\max_{zd_{j,k,t},zm_{j,k,t}} z_{j,k,t}=\left(\left(1-\theta_{j,k}^{zm}\right)\left(zd_{j,k,t}\right)^{\frac{\eta_{j,k}^{zm}-1}{\eta_{j,k}^{zm}}}+\left(\theta_{j,k}^{zm}\right)\left(zm_{j,k,t}\right)^{\frac{\eta_{j,k}^{zm}-1}{\eta_{j,k}^{zm}}}\right)^{\frac{\eta_{j,k}^{zm}}{\eta_{j,k}^{zm}-1}}$$

subject to:

$$p_{j,k,t}^{z}z_{j,k,t}\geq(1+\omega_{j,k,t}^{z}\tau_t^{gst})(1+\tau_{j,k,t}^{oit,z})p_{j,t}^{y}zd_{j,k,t}+(1+\omega_{j,k,t}^{z}\tau_t^{gst})(1+\tau_{j,k,t}^{oit,z})p_{j,t}^{m}zm_{j,k,t}$$

where: $\eta_{j,k}^{zm}$ is the elasticity of substitution; $0<\theta_{j,k}^{zm}<1$ is the CES weight for imported goods; $\omega_{j,k,t}^{z}$ is share of good k that is subject to the GST; and $\tau_{j,k,t}^{oit,z}$ is an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of intermediate goods and services.

Government spending

Government spending also has a two-tiered preference structure, although it has a different allocation mechanism for goods and varieties.

At the top-tier, the government is assumed to undertake spending with a fixed bundle of goods and services $g_{j,t}$ supplied by the distribution sector. Given the market price of each good $p^{g}_{j,t}$, total government spending is:

$$p^{g}_{t} g_{t} = \sum_{j} p^{g}_{j,t} g_{j,t} \tag{26}$$

Here p^{g}_{t} is the price of composite good purchased by the government, and real government expenditure g_{t} is assumed to be a CES aggregate of the bundle:

$$g_{t} = \left(\sum_{j} \theta^{g}_{j} \left(g_{j,t} \right)^{\frac{\eta^{g}-1}{\eta^{g}}} \right)^{\frac{\eta^{g}}{\eta^{g}-1}} \tag{27}$$

With $\eta^{g} > 0$ denotes the elasticity of substitution, and $0 < \theta^{g}_{j} < 1$ denotes the CES weight for variety j with $\sum_{j} \theta^{g}_{j} = 1$.

At the bottom-tier, the distribution sector optimally allocates funds $p^{g}_{j,t} g_{j,t}$ on behalf of the government to domestically produced $gd_{j,t}$ and imported $gm_{j,t}$ varieties of good j:

$$\max_{gd_{j,t}, gm_{jt}} g_{j,t} = \left(\left(1 - \theta^{gm}_{j}\right) \left(gd_{j,t}\right)^{\frac{\eta^{gm}_{j}-1}{\eta^{gm}_{j}}} + \left(\theta^{gm}_{j}\right) \left(gm_{j,t}\right)^{\frac{\eta^{gm}_{j}-1}{\eta^{gm}_{j}}} \right)^{\frac{\eta^{gm}_{j}}{\eta^{gm}_{j}-1}}$$

subject to:

$$p^{g}_{j,t} g_{j,t} \geq p^{y}_{j,t} gd_{j,t} + p^{m}_{j,t} gm_{j,t}$$

where: $\eta^{gm}_{j} > 0$ is the elasticity of substitution; and $0 < \theta^{gm}_{j} < 1$ is the CES weight for imported goods.

Goods and services tax

The Australian GST framework is summarised in Table 1, based on information from the Australian Taxation Office (ATO, 2022). There are three possible outcomes: the sector is ‘subject to GST’ if the goods and services sold by the sector is taxed at the GST rate and the sector is credited for the GST paid on its inputs; the sector is ‘GST free’ if the goods and services sold by the sector are not subject to

GST and the sector is credited for the GST paid on its inputs; and the sector is 'input-taxed' if the goods and services sold by the sector are not subject to GST and the sector is not credited for the GST paid on its inputs.

We capture this variation through coverage factors that are specific to the type of good and service and its stage of production. For example, if a sector falls under the 'subject to GST' category all GST paid on intermediate inputs and investment goods will be refunded by the government:

$$\omega^{i}_{j,k,t} = \omega^{z}_{j,k,t} = 0,$$

while the full statutory rate will apply to goods sold to the consumer:

$$\omega^{c}_{j,t} = 1.$$

However, the sectoral detail in the model is typically not at the level of individual goods and services where the coverage factors are zero or one, so we must use value-weighted shares of individual consumption, investment or intermediate goods to estimate the coverage factors that applies to the sectors in the model. These estimated coverage factors lie between zero and one.

Table 1: Taxation under different GST treatments

Treatment	Output subject to GST	Credits for GST on inputs
Subject to GST	Yes	Yes
GST-free	No	Yes
Input-taxed	No	No

Source: ATO (2022).

Given the statutory GST rate and the coverage factors, the total GST collected from the distribution sector is:

$$\begin{aligned} gst_t = {} & \tau^{gst}_t \sum_k p^y_{k,t} \left(\sum_a \sum_\ell \omega^c_{k,t} \left(1+\tau^{oit,c}_{k,t}\right) cd^{a,\ell}_{k,t} h^{a,\ell}_t + \sum_j \omega^i_{j,k,t} \left(1+\tau^{oit,i}_{j,k,t}\right) id_{j,k,t} + \sum_j \omega^z_{j,k,t} \left(1+\tau^{oit,z}_{j,k,t}\right) zd_{j,k,t} \right) \\ & + \tau^{gst}_t \sum_k p^m_{k,t} \left(\sum_a \sum_\ell \omega^c_{k,t} \left(1+\tau^{oit,c}_{k,t}\right) cm^{a,\ell}_{k,t} h^{a,\ell}_t + \sum_j \omega^i_{j,k,t} \left(1+\tau^{oit,i}_{j,k,t}\right) im_{j,k,t} + \sum_j \omega^z_{j,k,t} \left(1+\tau^{oit,z}_{j,k,t}\right) zm_{j,k,t} \right) \end{aligned}$$

Other indirect taxes

Given the effective ad-valorem tax rates for other indirect taxes, the revenue collected by the government is:

$$\begin{aligned} oit_t = {} & \sum_k p^y_{k,t} \left(\sum_a \sum_\ell \tau^{oit,c}_{k,t} cd^{a,\ell}_{k,t} h^{a,\ell}_t + \sum_j \tau^{oit,i}_{j,k,t} id_{j,k,t} + \sum_j \tau^{oit,z}_{j,k,t} zd_{j,k,t} \right) \\ & + \sum_k p^m_{k,t} \left(\sum_a \sum_\ell \tau^{oit,c}_{k,t} cm^{a,\ell}_{k,t} h^{a,\ell}_t + \sum_j \tau^{oit,i}_{j,k,t} im_{j,k,t} + \sum_j \tau^{oit,z}_{j,k,t} zm_{j,k,t} \right) \end{aligned}$$

Government

The government is defined as the 'general government sector'. For Version 1.0 of OLGA, detailed modelling of the government sector has been confined to Commonwealth government taxes and transfers, with rules of thumb for state and local taxes and government spending. Detailed modelling of state and local taxes including land tax and municipal rates will be incorporated in future model development. Similarly, detailed modelling of general government spending will be considered in a development module dedicated to government spending.

Revenue

The government raises revenue through various forms of taxation, with the total tax collected denoted tax_t. These taxes include personal income tax pit_t, corporate income tax cit_t, payroll tax prt_t, goods and services tax gst_t, other ad valorem taxes oit_t, and withholding taxes wt_t which are the sum of withholding tax on income earned by non-residents from investment in Australian government bonds and corporate bonds:

$$tax_t = pit_t + cit_t + prt_t + gst_t + oit_t + wt_t \tag{28}$$

$$pit_t = \sum_a \sum_\ell pit_t^{a,\ell} h_t^{a,\ell}$$

$$cit_t = \sum_j cit_{j,t}$$

$$prt_t = \sum_j prt_{j,t}$$

Payments

Spending

As noted above, the government is assumed to undertake spending on a fixed bundle of goods and services supplied by the distribution sector. This bundle gives the aggregate real government expenditure g_t at price p_t^g as per equations (26) and (27).

Following the broader literature, government spending does not enter the household utility function. Both the level of government spending and the utility it provides will be the subject of the government spending development module.

Transfers

Households can access the age pension (subject to means-testing) and other social transfers (see age pension and social transfer functions above). The total cost of the age pension is pen_t, while the total cost of other transfers is tr_t:

$$pen_t = \sum_a \sum_\ell pen_t^{a,\ell} h_t^{a,\ell}$$

$$tr_t = \sum_a \sum_\ell tr_t^{a,\ell} h_t^{a,\ell}$$

Government budget constraint

The government's flow budget constraint is:

$$(1+\gamma_t) b_{t+1}^g = p_t^g g_t + (1+r_t^g) b_t^g + pen_t + tr_t - tax_t \quad (29)$$

Where b_t^g is the level of government net debt at the beginning of period t.

In a model where households have perfect foresight, the government cannot incur unbounded liabilities or accumulate unbounded assets in the long run. To rule out this possibility we assume that the government credibly commits to a fiscal rule that ensures its debt to GDP ratio is stabilised at a given target within a finite period. In other words, the government must adopt a funding offset that yields a path of primary budget balances sufficient to achieve and stabilise debt at its target level by the specified date. This means that permanent increases in spending or reductions in tax revenue must be matched by permanent increases in other taxes or reductions in spending. Similarly, temporary increases to spending or reductions in tax revenue must be either financed by a temporary offset sufficient to stabilise debt at its original level, or a permanent increase in taxation or cuts to spending sufficient to fund the higher interest expenditure that comes with a higher debt level.

Financial sector

The financial sector allocates household savings to domestic and foreign assets. Under the small open economy and perfect capital market assumptions any shortfall in funds is sourced from the foreign sector, with the foreign investor being the marginal investor in all asset markets.

Household savings

Domestic households supply the following level of funds:

$$v_t^h = \sum_a \sum_\ell v_t^{a,\ell} h_t^{a,\ell} \quad (30)$$

Investment decisions are managed by a notional funds manager.

The allocation of household savings follows a nested structure. At the top tier, households are required to allocate a share of savings to ownership of dwellings with the remainder allocated to non-dwelling assets as described in detail below.

OLGA Version 1.0 assumes the Australian dwellings sector is fully owned by Australian households. This means the savings allocated to dwellings must equal the market value of the dwellings sector:

$$\pi_{dwe,t}^e v_t^h = v_{dwe,t}^c$$

Where $\pi_{dwe,t}^e > 0$ is the share of household savings allocated to dwellings. In this version of OLGA, houses are assumed to be owner-occupied so the income stream from housing is not taxed and there is no need to distinguish between equity holdings that return earnings via capital gain or dividends. This limitation will be addressed in a development module dedicated to the taxation of housing.

At the second tier, the remaining share of household savings $1-\pi^{e}_{dwe,t}>0$ is allocated to non-dwelling assets. These savings are distributed across a fixed-weight portfolio of domestic and foreign assets. The domestic component includes: domestic corporate and government bonds, with weights of $\pi^{bc}_{t}>0$ and $\pi^{bg}_{t}>0$ respectively; and equity in domestic firms in all non-dwelling sectors with a weight of $\pi^{ed}_{t}>0$ for firms that distribute earnings as fully franked dividends, and a weight of $\pi^{ec}_{t}>0$ for firms that distribute earnings as capital gains. The foreign component is assumed to be a composite of foreign assets (foreign bonds and equity) with weight equal to $\pi^{f}_{t}=(1-\pi^{bc}_{t}-\pi^{bg}_{t}-\pi^{ed}_{t}-\pi^{ec}_{t})>0$.

For Version 1.0 of OLGA we assume that the portfolio for household savings allocated to non-dwelling assets is fixed. This assumption will be relaxed in a development module dedicated to the taxation of savings.

Domestic government bonds

For simplicity we assume the government issues one period bonds. The assumed global required after tax rate of return for sovereign/government debt is r^{g*}_{t}. Foreign investors receiving income from bonds must pay withholding tax at the rate τ^{wt}_{t}. A country risk premium ρ^{g}_{t} implies the following global required before tax rate of return on Australian government bonds:

$$r^{g}_{t}=\frac{r^{g*}_{t}+\rho^{g}_{t}}{1-\tau^{wt}_{t}} \tag{31}$$

Domestic government bond holders who are subject to a marginal personal income tax rate τ^{pit}_{t} will earn the following after tax return:

$$(1-\tau^{pit}_{t})r^{g}_{t}=\frac{(1-\tau^{pit}_{t})(r^{g*}_{t}+\rho^{g}_{t})}{1-\tau^{wt}_{t}} \tag{32}$$

Based on the fixed asset allocation shares described above, the value of government bonds held domestically is:

$$b^{hg}_{t}=\left(1-\pi^{e}_{dwe,t}\right)\pi^{bg}_{t}v^{h}_{t} \tag{33}$$

This implies the following foreign holding of Australian government bonds:

$$b^{fg}_{t}=b^{g}_{t}-b^{hg}_{t} \tag{34}$$

and withholding tax revenue:

$$wt^{g}_{t}=\tau^{wt}_{t}r^{g}_{t}b^{fg}_{t}$$

Domestic corporate bonds

For simplicity we assume firms issue one period bonds. The global required return on corporate bonds is as follows:

$$r_t^c = \frac{r_t^{g*} + \rho_t^g + \rho_t^c}{1 - \tau_t^{wt}} \quad (35)$$

where: ρ_t^c is the risk premium on corporate bonds; and τ_t^{wt} is the withholding tax rate.

Domestic corporate bond holders who are subject to a marginal personal income tax rate τ_t^{pit} will earn the following after tax return:

$$(1 - \tau_t^{pit}) r_t^c = \frac{(1 - \tau_t^{pit})(r_t^{g*} + \rho_t^g + \rho_t^c)}{1 - \tau_t^{wt}} \quad (36)$$

Given the fixed asset allocation of shares described above the value of corporate bonds held domestically is:

$$b_t^{hc} = \left(1 - \pi_{dwe,t}^e\right) \pi_t^{bc} v_t^h \quad (37)$$

This implies the following foreign holding of Australian corporate bonds:

$$b_t^{fc} = b_t^c - b_t^{hc} \quad (38)$$

and withholding tax revenue:

$$wt_t^c = \tau_t^{wt} r_t^c b_t^{fc}$$

Domestic equity

The required rate of return for a foreign equity investor will depend on the way earnings are distributed. Foreign investors do not pay tax in Australia on capital gains earned on Australian assets which implies the following required rate of return to foreign equity holders when their return is realised as a capital gain:

$$r_t^{e1} = r_t^{g*} + \rho_t^g + \rho_t^c + \rho_t^e \quad (39)$$

where ρ_t^e is an equity premium.

In contrast, foreign investors must pay withholding tax τ_t^{wtd} on dividends, which implies the following required rate of return to foreign equity holders when their return is realised as a dividend:

$$r_t^{e2} = \frac{r_t^{g*} + \rho_t^g + \rho_t^c + \rho_t^e}{1 - \tau_t^{wtd}} > r_t^{e1} \quad (40)$$

Given $r_t^{e2} > r_t^{e1}$, the lowest cost foreign investor (aka the marginal foreign investor) is one that realises their return as a capital gain. This implies that the global required rate of return on domestic equity is:

$$r_t^e = r_t^{g*} + \rho_t^g + \rho_t^c + \rho_t^e \quad (41)$$

In this version of the model we do model the optimal allocation of funds between non-dwelling assets by assuming that households invest in firms that pay dividends as well as firms that return earnings as a capital gain according to an exogenous fixed split π_t^{ed} and π_t^{ec}. Domestic and foreign equity investors earn the same rate of return. It follows that a domestic investor, subject to marginal personal income tax rate τ_t^{pit}, who realises their return as a dividend will have the following after tax return:

$$\frac{(1-\tau_t^{pit})r_t^e}{(1-\tau_t^{cit})} = \frac{(1-\tau_t^{pit})(r_t^{g*} + \rho_t^g + \rho_t^c + \rho_t^e)}{1-\tau_t^{cit}} \quad (42)$$

while those that realise their return as a capital gain will have the following after-tax return:

$$(1-\tau_t^{pit}(1-\phi_t^{cgd}))r_t^e = (1-\tau_t^{pit}(1-\phi_t^{cgd}))(r_t^{g*} + \rho_t^g + \rho_t^c + \rho_t^e) \quad (43)$$

where ϕ_t^{cgd} is the capital gains discount.

Based on the asset allocation shares described above, the equity in Australian firms held domestically (including in the dwellings sector) is:

$$v_t^{hc} = (\pi_{dwe,t}^e + (1-\pi_{dwe,t}^e)(\pi_t^{ed} + \pi_t^{ec}))v_t^h \quad (44)$$

This implies the following foreign holding of equity in Australian firms:

$$v_t^{fc} = v_t^c - v_t^{hc} \quad (45)$$

Household's foreign investment

Finally, domestic investors are assumed to earn a before Australian tax rate of return of r_t^{f*} on the composite of foreign assets (bonds and equity):

$$r_t^{f*} = r_t^{g*} + \rho_t^{f*} \quad (46)$$

Where ρ_t^{f*} is the risk premium of the foreign portfolio over the sovereign borrowing rate.

Based on the asset allocation shares described above, foreign assets held domestically is:

$$v_t^f = (1-\pi_{dwe,t}^e)\pi_t^f v_t^h \quad (47)$$

Household's return on savings

The household's implied before personal income tax rate of return on savings is as follows:

$$r_t^h = \pi_{dwe,t}^e r_t^e + \left(1-\pi_{dwe,t}^e\right)\left[\frac{q_{j,t} + (1+\gamma_t)v_{j,t+1}^c - v_{j,t}^c}{v_{j,t}^c}\left(\pi_t^{ed} + \pi_t^{ec}\right) + r_t^c \pi_t^{bc} + r_t^g \pi_t^{bg} + r_t^{f*}\pi_t^f\right]$$

(48)

Because ownership of dwellings is exempted in Version 1.0 of OLGA, the rate of return on household savings for the purpose of calculating personal income tax payable is as follows:

$$\tilde{r}_t^h = \left(1-\pi_{dwe,t}^e\right)\left(\begin{array}{l}\sum_{j\neq dwe}\frac{d_{j,t}}{v_{j,t}^c}\frac{\pi_t^{ed}}{1-\tau_{j,t}^{cit}} + \sum_{j\neq dwe}\frac{(1+\gamma_t)v_{j,t+1}^c - v_{j,t}^c - nv_{j,t}^c}{v_{j,t}^c}\pi_t^{ec}(1-\phi_t^{cgd}) \\ + r_t^c\pi_t^{bc} + r_t^g\pi_t^{bg} + r_t^{f*}\pi_t^f\end{array}\right) \quad (49)$$

This reflects that dividends are grossed up to account for franking credits, while earnings returned via share buyback reflect a discounted capital gain. Returns from corporate bonds r_t^c, government bonds r_t^g and foreign assets r_t^{f*} are taxed at full value. As we do not model superannuation separately in this version of OLGA, there is no concessional treatment of savings through superannuation.

For simplicity households are assumed to turn over their savings every period. As such, they are subject to tax/credit on the capital gain/loss resulting from the change in asset prices in that period. Foreign investors are not subject to capital gains tax. Corporate bonds, government bonds and the household's foreign portfolio do not experience capital gain or loss because they are assumed to be one period securities. This means equity is the only household asset subject to price change.

Total foreign liabilities

The total foreign liabilities of the Australian economy and the related portfolio weights are then:

$$\begin{aligned} v_t^{f*} &= b_t^{fg} + b_t^{fc} + v_t^{fc} \\ \pi_t^{bg*} &= b_t^{fg} / v_t^{f*} \\ \pi_t^{bc*} &= b_t^{fc} / v_t^{f*} \\ \pi_t^{ec*} &= v_t^{fc} / v_t^{f*} \end{aligned} \quad (50)$$

where: $\pi_t^{bg*} > 0$ is the implied share of foreign liabilities accounted for by foreign holdings of Australian government bonds; $\pi_t^{bc*} > 0$ is the implied share of foreign liabilities accounted for by foreign holdings of Australian corporate bonds; and $\pi_t^{ec*} = (1-\pi_t^{bc*} - \pi_t^{bg*}) > 0$ is the implied share of foreign liabilities accounted for by foreign holdings of equity in Australian firms.

The implied after-tax rate of return on this portfolio is:

$$r_t^f = \left[(1-\tau_t^{wt})r_t^g\pi_t^{bg*} + (1-\tau_t^{wt})r_t^c\pi_t^{bc*} + r_t^e\pi_t^{ec*}\right] \quad (51)$$

Competitive Equilibrium

Given the demographic structure $h_t^{a,\ell}$, relative labour efficiency $\xi^{a,\ell}$, labour augmenting technical progress ξ_t , government policy (spending, transfers and tax), foreign prices $p_{j,t}^m$, the global required after tax rate of return for sovereign/government debt r_t^{g*} , risk premia and the initial distribution of household asset holdings $v_0^{a,\ell}$, a competitive equilibrium corresponds to a sequence of prices $\left\{p_{j,t}^y\right\}_{j=s}^J$ and a corresponding sequence of decisions by households $\left\{\left\{c_t^{a,\ell}, 1-N_t^{a,\ell}\right\}_{a=21}^{95}\right\}_{\ell=1}^5$ and firms $\left\{y_{j,t}, k_{j,t}, \tilde{n}_{j,t}, z_{j,t}\right\}_{j=s}^J$ that satisfy the following conditions:

(i) Households maximise intertemporal utility (1) subject to their budget constraint (7);

(ii) Firms maximise the market value of the firm (18) subject to their budget constraint (22);

(iii) The government's budget constraint (29) is satisfied;

(iv) Labour and capital markets clear;

(v) Goods markets clear:

$$y_{k,t} = \sum_a \sum_\ell cd_{k,t}^{a,\ell} h_t^{a,\ell} + \sum_j id_{j,k,t} + \sum_j zd_{j,k,t} + x_{k,t} \tag{51}$$

If the demographics and government policies are stabilised in the long term, then a balanced-growth path exists where, with the exception of real wages which grow at constant rate γ_∞^ξ , relative prices are stationary and all other variables (in per capita terms) are growing at the constant rate of γ_∞ .

Solution method

Following Auerbach and Kotlikoff (1987), we solve the model by using an improved Gauss-Seidel algorithm[6]. Intuitively, the Gauss-Seidel algorithm can be thought of as a Walrasian auction system that allocates resources until an economic equilibrium is reached.

The auctioneer in this system has no control over the interest rate or import prices, since our assumption that Australia is a small-open economy means these are set by the global market. But the auctioneer does set the price of goods and services and the aggregate wage, announcing each to a crowd of agents comprising firms, households, the foreign sector and the government. Based on this announcement, each agent proposes its own response. Firms in each sector determine the level of output and demand for capital, labour and intermediate goods that will maximise their market value. Households determine the labour supply and demand for consumption goods that will maximise their lifetime utility. The foreign sector determines a level of exports given their preferences and the announced prices. And the government determines how to balance its budget and allocate

6 See Ludwig (2007) and Heer and Maussner (2009).

government spending in light of the announced prices. All agents – firms, households, the foreign sector, and the government – submit their intended responses to the auctioneer. The auctioneer reviews these responses and continues to adjust the prices/wage until the goods and labour markets have cleared and the wage is equalised across all sectors. The system is competitive because neither the firms, the households, the foreign sector nor the government can on their own determine the prices/wage announced by the auctioneer. The auction therefore results in a competitive equilibrium.

An outline of the solution method is deferred to Appendix F. The model is implemented using Matlab, a commercial software package that is widely used across industry and academia. Typically, it takes around half an hour to solve the model for the steady states and the transition path.

Calibration

The model is calibrated to match Australian demographic, economic, government and financial data. Where possible the model's parameters are estimated directly using Australian data sources or reflect actual policy settings. Otherwise, parameters reflect consensus in the literature.

In Version 1.0 we assume that in the absence of policy change the economy is on the balanced growth path. Working toward that end the growth rates of the model's exogenous trends (that is, the population and labour augmenting technical progress) are constant. Similarly, exogenous lifecycle profiles for relative skill levels and survival probabilities are time-invariant.

When calibrating the model, it is necessary to make a decision about what historical data period the model is calibrated to. Due to data availability it is not always possible to calibrate all parts of the model to the same historical period. Version 1.0 of the model is designed to approximate the Australian economy around 2015-16, which is a period for which most of the data necessary to calibrate the model is available. Underlying this are the following set of key calibration assumptions:

(a) parameters governing the production side of the economy and preferences over different commodities are calibrated to the 2014-15 Australian Input-Output tables;

(b) macroeconomic ratios such as the capital-to-output ratio and the net foreign asset to GDP position and government expenditure and revenue to GDP ratios are generally calibrated to the average over the five-year period to 2015-16;

(c) survival probabilities, and therefore life expectancy, are calibrated to 2016-18 levels; and

(d) fiscal policy variables reflect policy settings as at 2016-17; and

(e) the relative earnings profiles and other household lifecycle variables are calibrated to the data in the 2015-16 Survey of Income and Housing (SIH) published by Australian Bureau of Statistics (ABS).

Fiscal policy variables that are expressed in dollar terms (such as income tax thresholds) are normalised to have the same relationship to GDP per capita as they did in the 2015-16 financial year. Along the balanced growth path, all value policy variables grow at the rate of GDP per capita.

Household

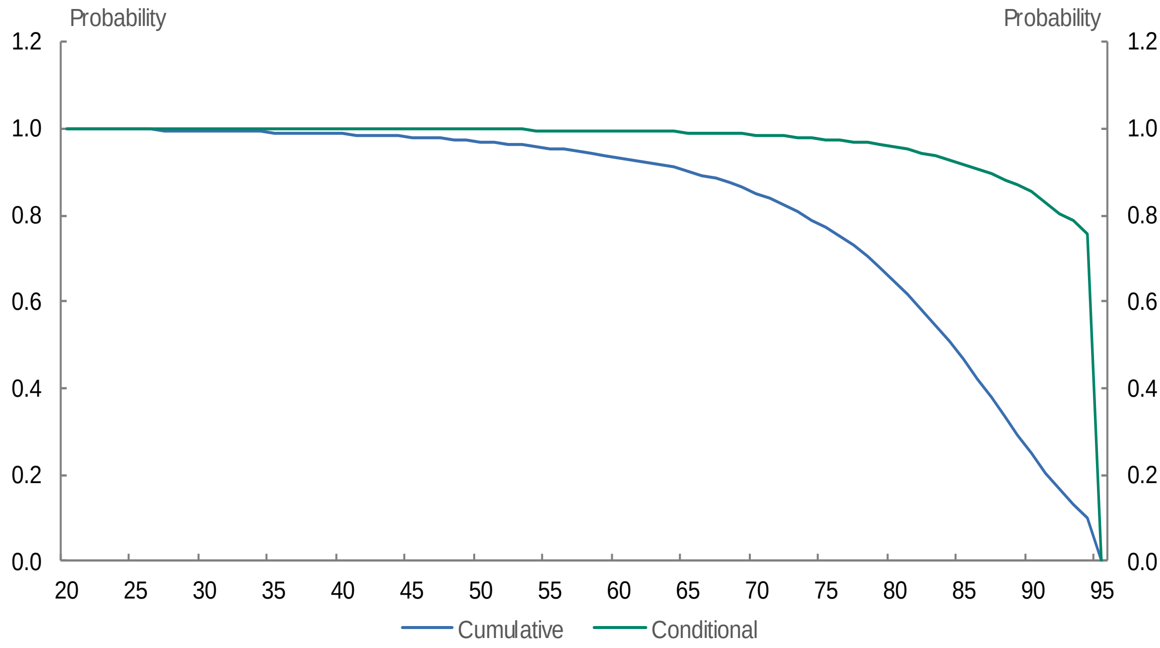
Demographic structure

Population growth

Version 1.0 of OLGA assumes a stationary age distribution of the population and a constant population growth rate. This reflects a constant growth rate of the population of 21-year-olds and time-invariant survival probabilities.

The growth rate of the population of 21-year-olds $\gamma^{h,21}$ is assumed to be 1.5 per cent per annum. Chart 1 plots the model's conditional ψ^a and cumulative Ψ^a survival probability by age. Survival probabilities are calculated using ABS Life Tables for Australia (2019a). The conditional survival probability is calculated as one minus the proportion of people dying between age a and $a+1$.

Chart 1: Survival probability by age



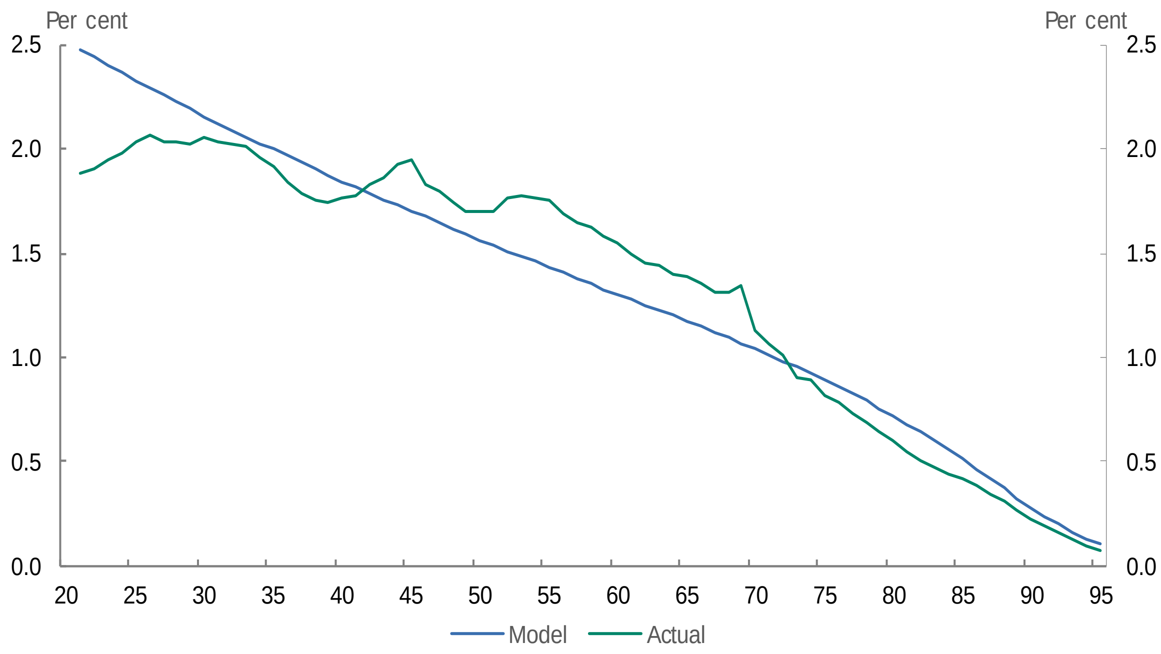
Source: Authors' calculation based on ABS (2019a) data.

Given that the survival probabilities are time-invariant, it follows that the age distribution is also time-invariant and that the total population grows at the same rate as the population of 21-year-olds (that is, $\gamma^{h,21} = \gamma^h$):

$$H_{t+1} = \sum_{a=21}^{95} H_{t+1}^a = \sum_{a=21}^{95} H_t^a (1+\gamma^{h,21}) = H_t\,(1+\gamma^{h,21}) = H_t\,(1+\gamma^h)$$

Chart 2 plots the actual and model implied age distribution of the population for the base year. The stationary distribution generated by the model overstates the share of the population under 30, while understating the share of the population between 40 and 70. This reflects the fact that the model does not incorporate immigration and assumes a time-invariant survival rate.

Chart 2: Age distribution of the population



Source: Authors' calculation based on ABS (2019a) and ABS (2020a).

Skill and technical progress

The households' earning ability and implied relative labour efficiency $\xi^{a,\ell}$ are calibrated using cross-sectional earnings data from SIH (ABS, 2017). For each income quintile, earning ability is estimated using the hourly wage rate, defined as gross labour income divided by total hours worked. To eliminate the effect of extreme outliers, we use the median of data from SIH to estimate the lifecycle profiles. These profiles are then rescaled, so that the population-weighted mean of each lifecycle profile matches the population-weighted mean of data from the SIH for each skill type.

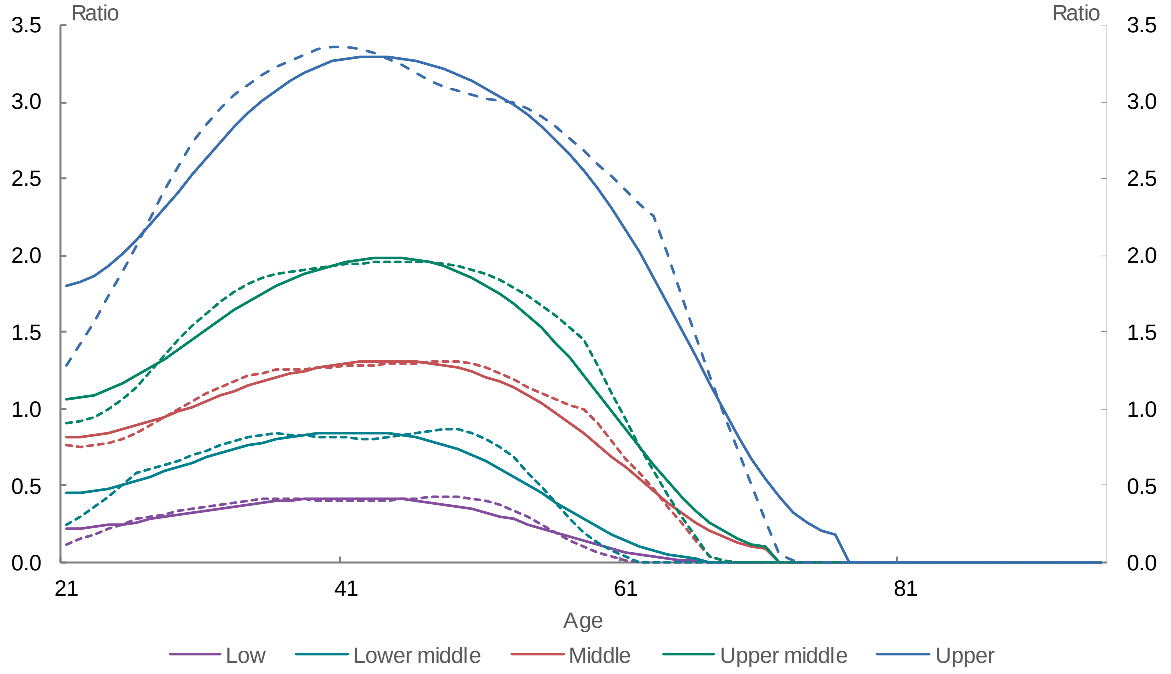
Because cross-sectional data suggests that the earning ability of the lowest quintile is effectively zero, we have assumed it is half that of the second lowest quintile (lower middle). A robust spline smoothing function is then used to provide the smoothed profiles used in the model. Chart 3 shows the fit of the estimated profiles to the actual cross-sectional earnings data.

The efficiency profiles are normalised, such that the sum of efficiency by age and skill type, weighted by respective population share, equals the level of labour augmenting technical progress:

$$\sum_a \sum_\ell \xi_t^{a,\ell} h_t^{a,\ell} / \sum_a \sum_\ell h_t^{a,\ell} = \xi_t$$

Consistent with the methodology used in the 2021 Intergenerational Report (Commonwealth of Australia, 2021b), the growth rate of labour-augmenting technical progress γ^ξ is assumed to be 1.5 per cent per annum. This implies a trend GDP growth rate γ of around 3 per cent per annum.

Chart 3: Relative labour efficiency by age and skill



Source: Authors' calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Preferences

Instantaneous utility

Following the Dynamic Stochastic General Equilibrium (DSGE) literature (for example, King, Plosser and Rebelo (1988)), we set the coefficient of relative risk aversion $\sigma = 2$ for all households. This implies an intertemporal elasticity of substitution that is equal to 1/2. Similar to Fehr (2000) and Kudrna et al. (2015), we have calibrated α for each skill type to match data on lifetime labour supply reported in SIH (ABS, 2017). All these parameters are reported in Table 2 below.

Table 2: Parameter values for the utility function

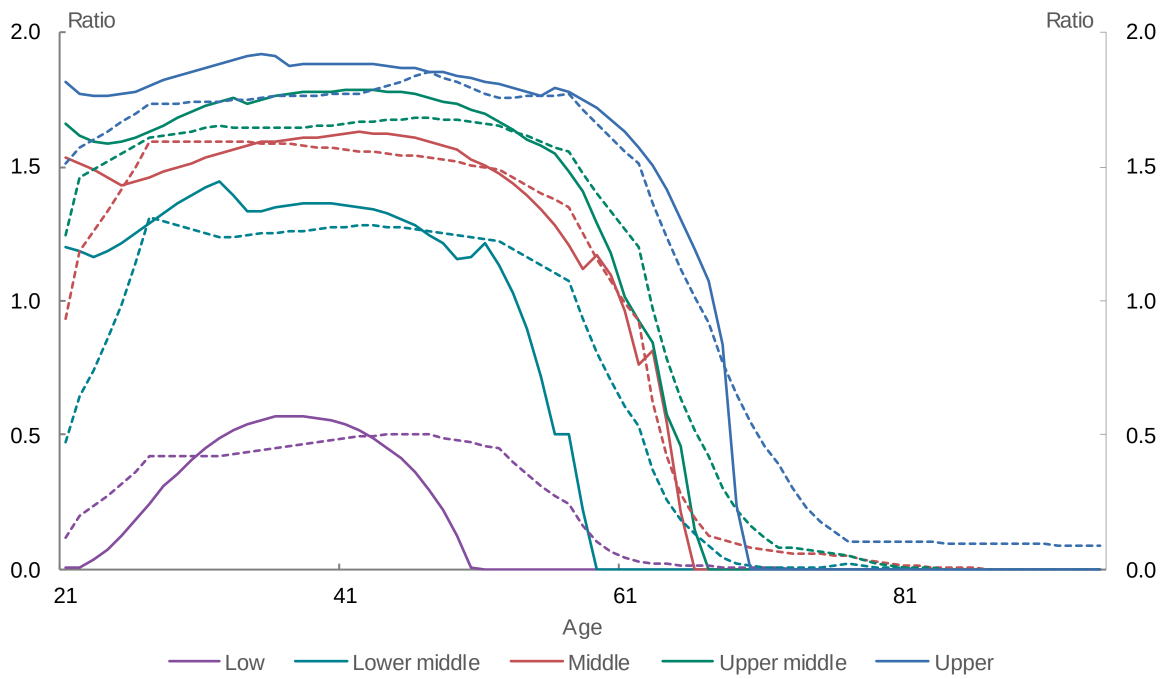
Parameter	Low	Lower-middle	Middle	Upper-middle	High	All
β						0.99
σ						2
α	0.5	0.6	0.65	0.7	0.75	
ϕ_1			5	10	20	
ϕ_2			0.02	0.02	0.02	

Consistent with this calibration, we assume that a typical adult Australian, after accounting for public holidays, personal and recreational leave, is available to work 5 days a week, for 45 weeks per year.

Per workday, the household is assumed to have 12 hours available for work and leisure. When combined this implies an annual time endowment of 45 x 5 x 12 = 2700 hours.

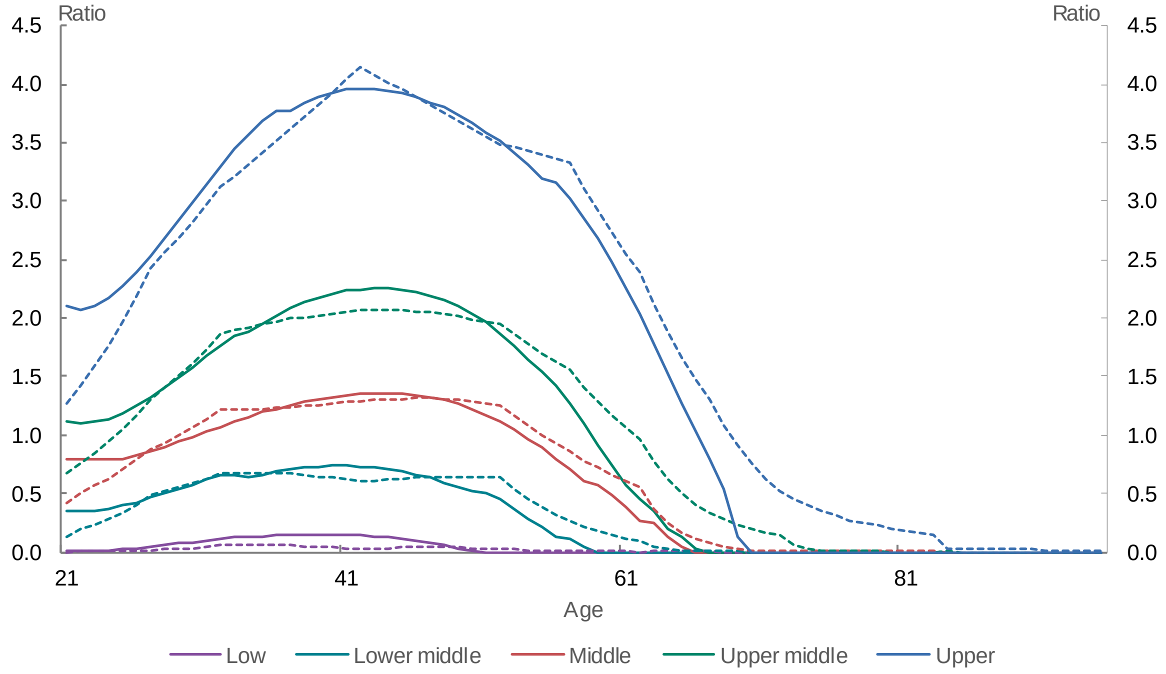
Charts 4 and 5 suggest our calibration captures labour supply profiles of workers 30 years and older well, while it tends to overstate the labour supply of workers below 30 years of age. There are kinks in the modelled labour supply profiles which are not apparent in the data. These kinks reflect discrete changes in marginal tax rates induced by the progressive personal income tax schedule. Other researchers (see for example, Kudrna and Tran (2018)) avoid this issue by approximating the tax schedule using a smooth income tax function. This approach has not been adopted here because it potentially understates the effects of changes in the progressive tax schedule.

Chart 4: Household labour supply by age and skill



Source: Authors' calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Chart 5: Household labour earnings by age and skill



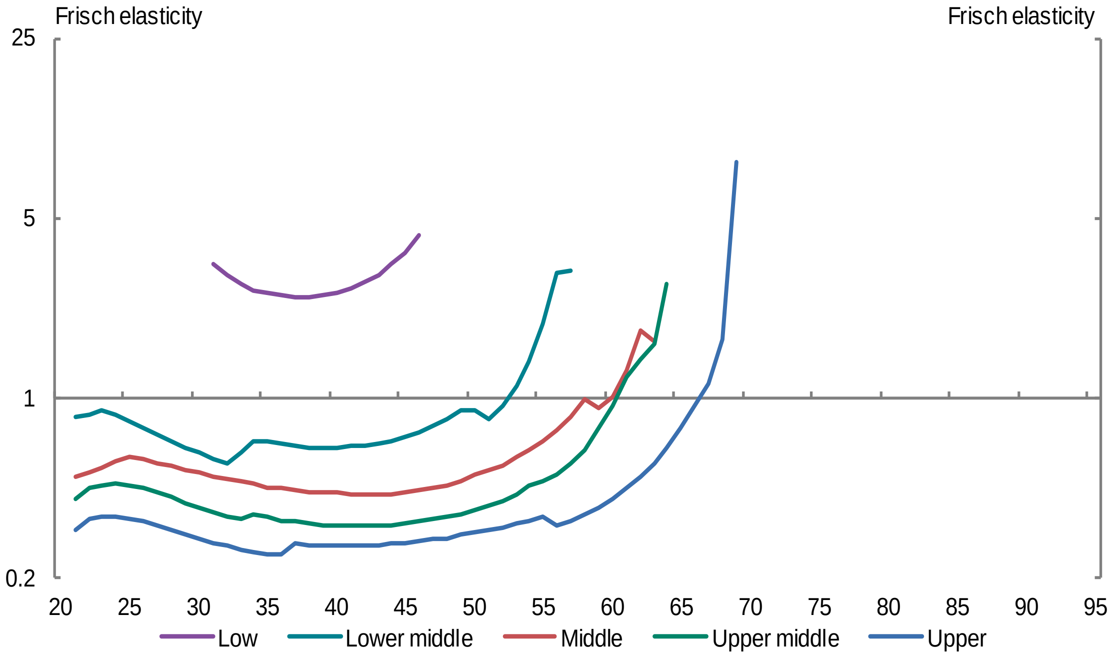
Source: Authors' calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Given α^{ℓ}, $N^{a,\ell}$ and σ , there is an implied Frisch elasticity of labour supply:

$$\eta^{a,\ell} = \frac{1-N^{a,\ell}}{N^{a,\ell}}\left[\frac{1-\alpha^{\ell}(1-\sigma)}{\sigma}\right]$$

The relatively low Frisch elasticities generated by the calibration methodology are consistent with the micro-econometric literature surveyed by Keane (2011), which suggests that workers have relatively low elasticities during their prime earning years (see Chart 6).

Chart 6: Implied Frisch elasticities by age and skill



Source: Authors' calculation.

Household savings and bequests

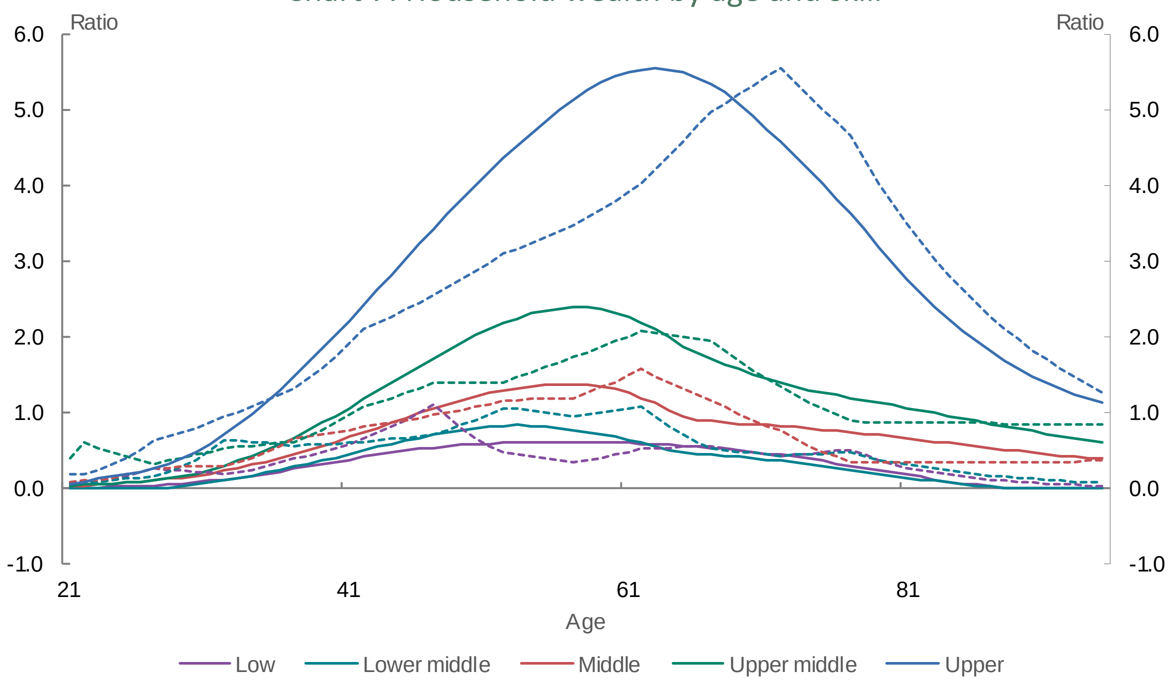
The aggregation of individual lifecycle profiles of household savings determines the steady state rate of national saving. The accumulation of household savings is jointly determined by the discount factor and the 'warm glow' motive to leave intended bequests.

Following the literature households have a common discount factor assumed to be $\beta = 0.99$. Given the calibrated consumption to GDP ratio this ensures the economy's initial net foreign liability position is consistent with the data at roughly 50 per cent of GDP (see Table 18 below for details).

Following the empirical study of Fink and Redaelli (2005), we have assumed that only households of the high, upper middle and middle skills accumulate assets over their lifetime with the intention of leaving a bequest. For those household types, ϕ_1 is calibrated to match their end-of-life savings in actual data, and ϕ_2 is calibrated to have a total bequest-to-GDP ratio of around 3 per cent. While this is only around half the level found by the Productivity Commission (2021), the model currently ignores the value of land used for dwellings which accounts around half of the wealth held by households. This means calibrated annual bequests are consistent with the findings of the Productivity Commission of around 1 per cent of total household wealth. These parameters are reported in Table 2.

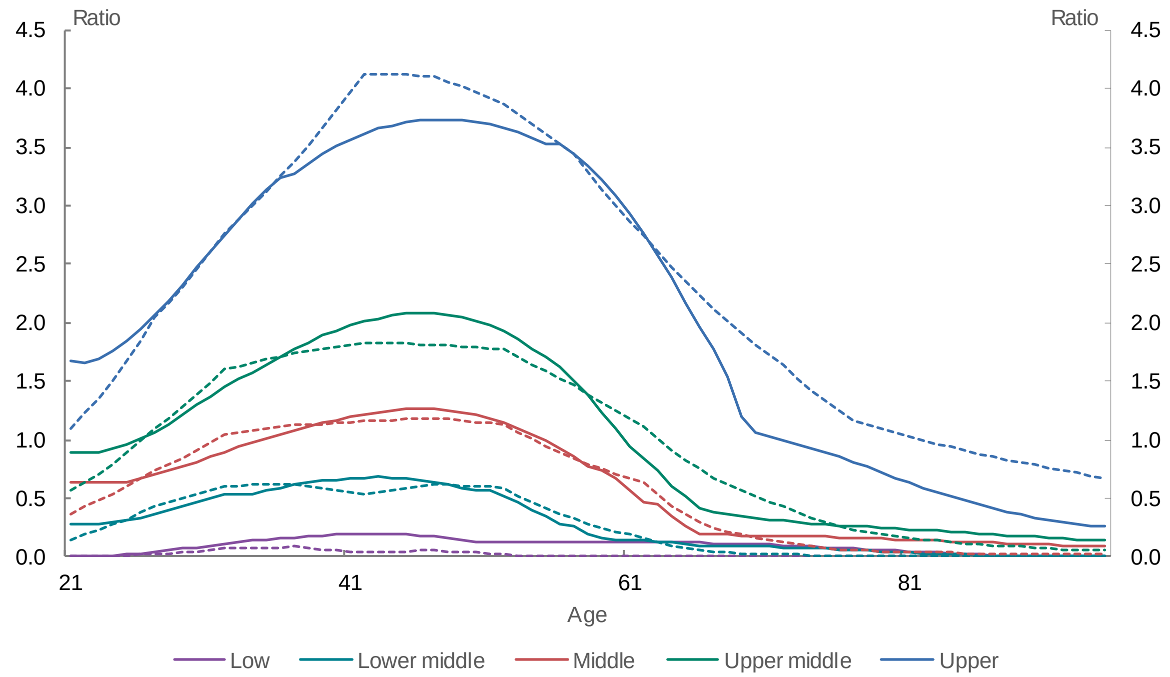
As Chart 7 shows, the model's implied distribution of household wealth does a reasonably good job in tracking the cross-sectional distribution of wealth. Given this the model also matches well the cross-sectional distribution of gross household income (see Chart 8).

Chart 7: Household wealth by age and skill



Source: Authors' calculations based on SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Chart 8: Gross household income by age and skill



Source: Authors' calculations based on SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Production sector

Version 1.0 of OLGA includes three goods sectors: Agriculture (AGR), Mining (MIN), and Manufacturing (MAN); and four services sectors: Utilities (UTL), Construction (CST), Services (SRV), and Dwellings (DWE). All sectors produce output that is exported and all sectors except for dwellings faces competing imports. Appendix G shows the concordance with the ABS Input-Output Industry Group.

Production technology

Primary factor and intermediate shares

We use the ABS (2018a) Input-Output (IO) table to calibrate the CES weights for the production and distribution sectors. The Input-Output table provides information about the supply and use of factors and products, and the inter-relationships between sectors. Table 3 recasts the broader 114 sector IO table reported by the ABS into a seven sector version consistent with the sectoral aggregation above.

Table 3: Australian Input-Output table (2014-15) per cent GDP

		Intermediate use								Final use					Total use
	Supply/Use	AGR	MIN	MAN	UTL	CST	SRV	DWE	Total	Household consumption	Government consumption	Investment	Exports	Total	
Intermediate	AGR	0.9	0.0	2.2	0.0	0.0	0.4	0.0	3.6	0.5	0.0	0.3	1.0	1.9	5.4
	MIN	0.0	1.0	2.7	0.2	0.2	0.5	0.0	4.6	0.2	0.0	1.1	7.5	8.8	13.4
	MAN	0.7	1.2	5.1	0.3	4.9	6.5	0.1	18.7	8.0	0.5	4.5	5.3	18.3	37.0
	UTL	0.1	0.2	0.5	1.5	0.2	1.3	0.0	3.7	1.5	0.1	0.4	0.0	2.0	5.7
	CST	0.2	0.7	0.2	0.2	7.8	2.2	0.6	11.9	0.1	0.0	14.2	0.1	14.3	26.2
	SRV	1.0	2.4	4.1	1.0	4.8	34.9	2.0	50.2	30.7	17.5	4.4	5.1	57.7	107.9
	DWE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.6	0.0	0.0	0.1	11.7	11.7
	Total intermediate	2.9	5.5	14.7	3.1	18.0	45.8	2.6	92.7	52.6	18.1	24.9	19.0	114.7	207.4
Factor inputs	Compensation of employees	0.8	1.8	4.0	0.8	5.3	39.9	0.0	52.7						52.7
	Gross operating surplus	1.5	4.8	2.1	1.6	2.5	16.6	8.2	37.3						37.3
	Total factor income	2.3	6.6	6.1	2.4	7.9	56.5	8.2	90.0						90.0
	Taxes less subsidies on production	0.1	0.1	0.2	0.2	0.2	1.8	0.7	3.2						3.2
	Gross value added	2.4	6.7	6.3	2.6	8.1	58.2	8.9	93.2						93.2
	Taxes on products	0.0	0.0	0.1	0.0	0.1	0.6	0.1	0.9	4.0		1.8	0.0	5.8	6.8
	Gross domestic product	2.4	6.7	6.4	2.6	8.1	58.8	9.0	94.2	4.0		1.8	0.0	5.8	100.0
	Gross value of production	5.3	12.2	21.1	5.7	26.1	104.1	11.6	185.9						185.9
Rest of world	Competing imports	0.1	1.2	15.9	0.0	0.1	3.3	0.0	20.5						20.5
	Total Supply	**5.4**	**13.4**	**37.0**	**5.7**	**26.2**	**107.9**	**11.7**	**207.4**						**207.4**

Source: Authors' calculation based on ABS (2018a) data.

Producer and import prices are normalised to unity in the initial steady state, $p^{y}_{j,0} = 1, p^{m}_{j,0} = 1$ for all j The scaling term in the production function $\lambda_{j,0}$ is calibrated to ensure the gross domestic output (GDP) per worker is one in the initial steady state. Given these assumptions the initial steady state prices of intermediate goods and services $p^{z}_{j,0}$, the CES weights in the production sector are estimated using the factor input and expenditure cost shares reported in Table 3. Further details on estimation are provided at the end of Appendix C.

Table 4 reports the CES weights for the primary factor and intermediate inputs implied by the purple and red blocks of Table 3.

Table 4: Production weights

Parameter	Consistent with cost share: j =						
	AGR	MIN	MAN	UTL	CST	SRV	DWE
θ^{ym}_{j}	0.1590	0.1489	0.1934	0.1502	0.2062	0.3900	0.0000
θ^{yk}_{j}	0.2873	0.3951	0.1006	0.2895	0.0977	0.1620	0.7559
θ^{yz}_{j}	0.5537	0.4560	0.7060	0.5604	0.6961	0.4480	0.2421

Source: Authors' calculation based on ABS (2018a) data.

Factor substitution

The elasticity of substitution between capital, labour and intermediate inputs η^{y}_{j} is assumed to be 0.5 for all j consistent with estimates of the elasticity of substitution between capital and labour reported by Hutchings and Kouparitsas (2012).

Investment/adjustment cost parameters

The industry capital depreciation rates for the seven sectors respectively, as reported in Table 5 are calibrated based on ABS (2021a).

Table 5: Capital depreciation rates

Parameter	j =						
	AGR	MIN	MAN	UTL	CST	SRV	DWE
$\delta_{j,0}$	0.093	0.066	0.107	0.040	0.109	0.065	0.030

Source: Authors' calculation based on ABS (2021a) data.

Following the broader DSGE literature the parameters for capital adjustment costs ζ^{k}_{j} are set to 2.5 for all sectors. Mendoza and Tesar (1998 and 2005) argue that this value is consistent with the average rate of convergence to the long-run balanced growth path estimated by Barro and Sala-i-Martin (2004).

Rest of the world

Foreign demand shares and elasticity

The level of foreign aggregate demand is normalised to be 1 for all sectors. The CES weights for Australian varieties in foreign demand θ_j^{cm*} for all sectors are calibrated using data on global exports from the GTAP Data Base (Aguiar et al. (2019)). This is consistent with the methodology used in multi-country models (for example, Hertel (1997); McKibbin and Wilcoxen (1998)). Taking into account required net export to GDP ratio to stabilise net foreign assets as a ratio to GDP and θ_j^{cm*}, the CES weights for all goods in foreign aggregate demand θ_j^{c*} are calibrated to match the distribution of exports reported in blue block of Table 3. Table 6 shows the final calibrated values of $x_{j,0}$ as a per cent of GDP.

Consistent with the elasticity of substitution between different commodities for domestic households and firms, the elasticity of substitution between different types of goods η^{c*} is set to 0.5. This reflects that consumers are relatively reluctant to substitute between commodities. The elasticities of substitution between varieties of the same good from different exporters η_j^{cm*} are based on those from the GTAP Data Base. These are also presented in Table 6.

Table 6: Export demand parameters

Parameter	j =						
	AGR	MIN	MAN	UTL	CST	SRV	DWE
θ_j^{cm*}	0.082	0.265	0.038	0.028	0.001	0.017	0.001
η_j^{cm*}	6.4	6.3	7.3	5.6	3.8	3.8	3.8
$x_{j,0}$	2.24%	8.61%	6.13%	0.03%	0.07%	5.92%	0.14%

Source: Authors' calculations based on Aguiar et al. (2019) and ABS (2018a). The unit of $x_{j,0}$ is % of GDP.

Distribution sector

Composite goods

Expenditure shares

The CES weights in the distribution sector composite goods and services functions are calibrated to match final expenditure shares reported in the ABS (2018a) IO table. In particular, the weights for intermediate, consumption, investment, and government consumption are calibrated to match the expenditure shares implied by the purple (intermediate only), green, grey, and orange blocks of Table 3. Further details on calibration are provided at the end of Appendix E. The expenditure shares implied by these data are summarised in Table 7.

The ABS Input-Output tables do not provide investment expenditure by purchasing industry (this means there is only one private investment column in the Input-Output table). To better capture the investment goods purchased by different sectors the investment goods sold by each industry have been allocated across the purchasing industries by also using additional data on private investment by capital type and two-digit industry from the ABS National Accounts 5204.0. This allocation has also been used to allocate the GST paid on investment goods and services to different purchasing industries.

Table 7: Expenditure weights

Parameter	Consistent with cost share: j =						
	AGR	MIN	MAN	UTL	CST	SRV	DWE
θ_j^c	0.0105	0.0037	0.1528	0.0287	0.0010	0.5835	0.2198
$\theta_{AGR,j}^i$	0.2216	0.0000	0.3971	0.0000	0.1744	0.2069	0.0000
$\theta_{MIN,j}^i$	0.0000	0.0506	0.0709	0.0000	0.7716	0.1070	0.0000
$\theta_{MAN,j}^i$	0.0000	0.0000	0.3523	0.0000	0.1997	0.4479	0.0000
$\theta_{UTL,j}^i$	0.0000	0.0000	0.1678	0.0000	0.6821	0.1501	0.0000
$\theta_{CST,j}^i$	0.0000	0.0000	0.5547	0.0000	0.1048	0.3405	0.0000
$\theta_{SRV,j}^i$	0.0000	0.0000	0.3102	0.0000	0.4160	0.2738	0.0000
$\theta_{DWE,j}^i$	0.0000	0.0000	0.0000	0.0000	0.9153	0.0847	0.0000
$\theta_{AGR,j}^z$	0.3134	0.0060	0.2476	0.0431	0.0558	0.3342	0.0000
$\theta_{MIN,j}^z$	0.0044	0.1767	0.2156	0.0341	0.1318	0.4373	0.0000
$\theta_{MAN,j}^z$	0.1477	0.1827	0.3436	0.0334	0.0112	0.2814	0.0000
$\theta_{UTL,j}^z$	0.0006	0.0522	0.0821	0.4704	0.0737	0.3209	0.0000
$\theta_{CST,j}^z$	0.0025	0.0128	0.2747	0.0095	0.4346	0.2659	0.0000
$\theta_{SRV,j}^z$	0.0091	0.0115	0.1420	0.0282	0.0481	0.7612	0.0000
$\theta_{DWE,j}^z$	0.0000	0.0070	0.0253	0.0047	0.2144	0.7486	0.0000
θ_j^g	0.0014	0.0004	0.0261	0.0036	0.0007	0.9670	0.0008

Source: Authors' calculation based on ABS (2018a) and ABS (2021a) data.

Substitution of varieties

For the distribution sector, the elasticities of substitution in the basket of goods for the composite consumption, investment, intermediate inputs, and government spending functions ($\eta^c, \eta^i, \eta^z, \eta^g$) are set to 0.5.

Import demand

Import shares

The allocation of expenditure across domestic and imported alternatives follows the same methodology as the composite goods and services. The added complication is that prices must be adjusted for GST and other ad valorem taxes. The CES weights are derived from the cost shares of imported goods. The cost shares of imported goods are also reported in the ABS (2018a) as a supplement to the IO table. Table 9 reports the seven-sector version of that table, which adopts the same colour coding as Table 3. The expenditure cost shares implied by Tables 3, 8 and 9.

Table 8: Allocation of Imports (per cent of GDP)

	Intermediate use								Final use					Total uses
	AGR	MIN	MAN	UTL	CST	SRV	DWE	Total	Household consumption	Government consumption	Investment	Exports	Total	
AGR	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0		0.0	0.1
MIN	0.0	0.0	0.9	0.0	0.0	0.2	0.0	1.1	0.0	0.0	0.0		0.0	1.2
MAN	0.3	0.6	2.3	0.2	1.7	3.1	0.0	8.3	4.1	0.2	3.3		7.6	15.9
UTL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0
CST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1		0.1	0.1
SRV	0.0	0.1	0.1	0.0	0.2	1.4	0.0	1.9	1.3	0.0	0.1		1.4	3.3
DWE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0
Total	0.4	0.8	3.4	0.2	1.9	4.6	0.0	11.3	5.5	0.2	3.4		9.1	20.5

Source: Authors' calculation based on ABS (2018a) data.

Table 9: Import weights

Parameter	Consistent with cost share: j =						
	AGR	MIN	MAN	UTL	CST	SRV	DWE
θ^{cm}_{j}	0.0791	0.1593	0.5050	0.0008	0.0000	0.0433	0.0021
$\theta^{im}_{AGR,j}$ $\theta^{im}_{MIN,j}$ $\theta^{im}_{MAN,j}$ $\theta^{im}_{UTL,j}$ $\theta^{im}_{CST,j}$ $\theta^{im}_{SRV,j}$ $\theta^{im}_{DWE,j}$	0.0200	0.000	0.7208	0.0000	0.0050	0.0143	0.0000
$\theta^{zm}_{AGR,j}$	0.0091	0.0272	0.4463	0.0012	0.0004	0.0250	0.0000
$\theta^{zm}_{MIN,j}$	0.0152	0.0355	0.5346	0.0006	0.0004	0.0450	0.0000
$\theta^{zm}_{MAN,j}$	0.0139	0.3427	0.4559	0.0005	0.0010	0.0319	0.0000
$\theta^{zm}_{UTL,j}$	0.0620	0.0759	0.6278	0.0006	0.0002	0.0262	0.0000
$\theta^{zm}_{CST,j}$	0.0288	0.0134	0.3514	0.0010	0.0001	0.0402	0.0000
$\theta^{zm}_{SRV,j}$	0.0403	0.2858	0.4775	0.0007	0.0003	0.0391	0.0000
$\theta^{zm}_{DWE,j}$	0.0000	0.0023	0.3177	0.0000	0.0007	0.0107	0.0000
θ^{gm}_{j}	0.0000	0.0000	0.4369	0.0000	0.0000	0.0000	0.0000

Source: Authors' calculation based on ABS (2018a) data.

Import substitution

The elasticities of substitution between domestic and imported varieties $\eta^{cm}_{j}, \eta^{im}_{j,k}, \eta^{zm}_{j,k}, \eta^{gm}_{j}$ are consistent with the parameterisation of the GTAP model. In the GTAP model, the elasticity of substitution between domestic and imported varieties for a given commodity is set to half the value of the substitution between varieties of the same commodity from different exporters. We take these elasticities and aggregate them into the sectors in OLGA using each commodity's weight within that sector. The resulting elasticities are shown in Table 10.

Table 10: Import demand elasticities

Parameter	$j =$ AGR	MIN	MAN	UTL	CST	SRV	DWE
η_j^{cm}	1.9	17.1	2.9	2.8	1.9	1.9	1.9
$\eta_{j,k}^{im}, \eta_{j,k}^{zm}$	2.5	3.8	3.4	2.8	1.9	1.9	1.9
η_j^{gm}	2.2	0.9	2.0	2.8	1.9	1.9	1.9

Source: Authors' calculations based on Aguiar et al. (2019).

Government

The key aim of the calibration of the government sector in OLGA is to match Commonwealth government revenue, expenditure, and net debt levels. However, on the revenue side the model also incorporates state payroll taxes. In addition, a significant portion of Commonwealth revenue is usually provided to state governments (in particular, GST revenue). Therefore, government expenditure in the model can be better thought of as being calibrated to expenditure funded by Commonwealth revenue (and payroll tax) rather than expenditure directly undertaken by the Commonwealth government.

Taxation

Calibrated tax revenues are generally based on the ABS Taxation Revenue (ABS, 2021b) averaged over the 5-year period to 2015-16.

Personal (individuals) income tax

All marginal rates, thresholds, offsets, and the capital gains discount are calibrated to policy for the 2017-18 financial year (see Table 11 and Chart 9 for further details).

Table 11: Personal income tax schedule

	Income threshold	Marginal tax rate*
J	$\overline{yh}_{j,0}$	$\tau_{j,0}^{pit}$
0	0	0
1	$18,200	0.2*
2	$37,000	0.345*
3	$87,000	0.39*
4	$180,000	0.47*

* Includes the 2 per cent Medicare Levy, except for the first marginal rate which includes half the levy.

The capital gains discount for domestic equity ϕ_0^{cgd} is set to 0.5, consistent with current tax legislation.

The model baseline calibration includes two personal income tax offsets: the Low-Income Tax Offset (LITO); and the Seniors and Pensioners Tax Offset (SAPTO). Table 12 presents the parameters for each of the offsets. Consistent with the legislation, the SAPTO is exhausted before the LITO is applied. While this does not matter for the final tax liability, it matters for determining the actual effective marginal tax rate that an individual household faces. Because the model's steady state tax policy settings are calibrated to the 2017-18 financial year, we do not include more recent policies such as the low- and middle-income tax offset (LMITO) in the baseline calibration.

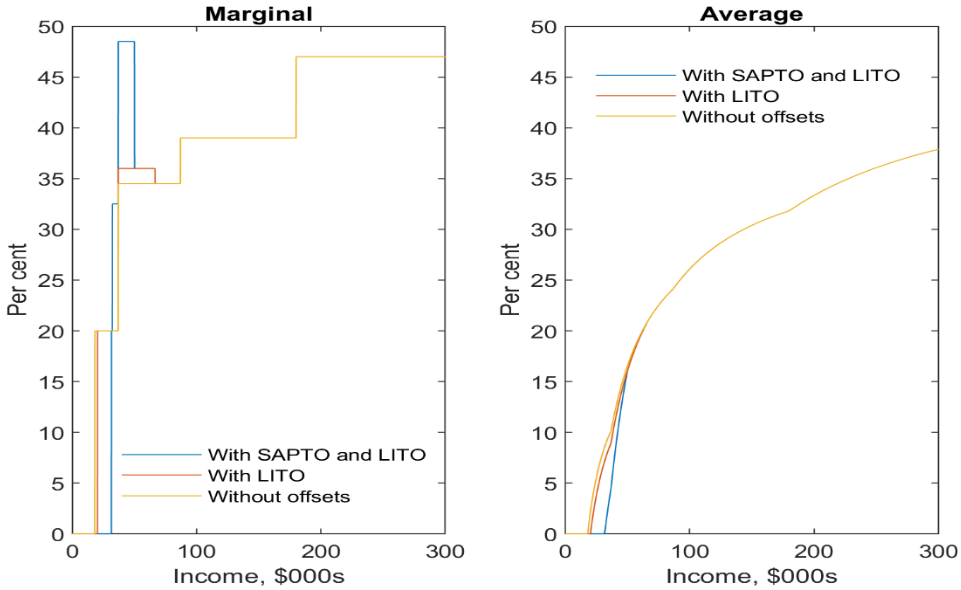
Table 12: Personal income tax offsets

Offset	Maximum offset	Taper rate	Lower income threshold	Upper income threshold
	$offsetmax_{j,0}$	$\omega^{o}_{j,0}$	$\overline{yh}^{o}_{1,j,0}$	$\overline{yh}^{o}_{2,j,0}$
SAPTO	$2230	0.125	$32,279	$50,119
LITO	$445	0.015	$37,000	$66,667

The tax schedules resulting from the combination of the personal income tax schedule and the personal income tax offsets are shown in Chart 9.

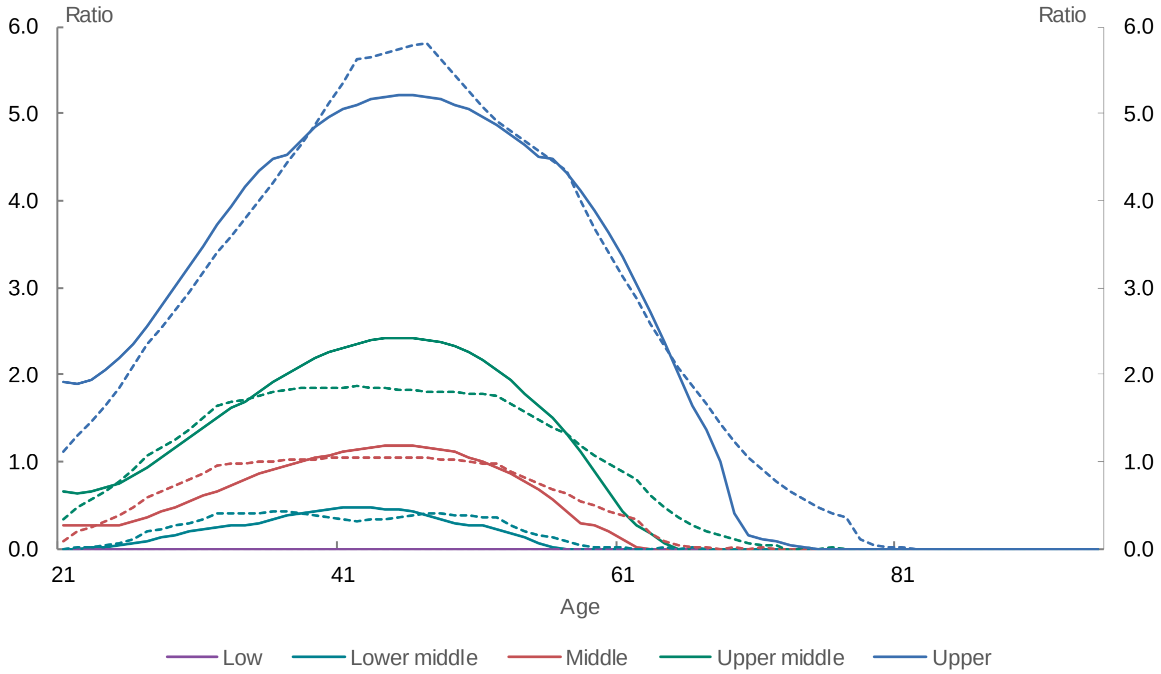
We calibrate the ratio of income tax deductions/exemptions $dt_t^{a,\ell}$ for each household type to match the lifecycle profile of tax liabilities (see Chart 10). The deductions/exemption ratio is 0.50 for the lowest skill type, 0.1 for the lower middle and middle skill types; and 0.12 for the upper middle and upper skill types.

Chart 9: Personal income tax rates



Note: The 'With SAPTO and LITO' tax schedule applies to those meeting the age pension eligibility age criteria, and the 'With LITO' schedule applies to everybody else.

Chart 10: Personal income tax liabilities by age and skill



Source: authors' calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Given the calibration, the average effective personal income tax rate τ_0^{pit} is 19.9 per cent and personal income tax revenue is around 13 per cent of GDP. This is broadly consistent with tax revenue from individual income tax, superannuation fund income tax and fringe benefit tax revenue reported in the ABS Taxation Revenue statistics (ABS 2021b).[7]

Corporate tax

Version 1.0 of OLGA matches a statutory corporate income tax rate τ_0^{cit} of 30%. Because there is a single representative firm in each sector, we cannot explicitly model the legislated lower tax rates for small and medium-sized businesses.

Under current legislation, interest on corporate debt is fully deductible for CIT purposes. While the model allows for an investment allowance, it is generally not applicable under current legislation so $\phi_{j,0}^{i} = 0$ for all sectors.

Conditional on the above assumptions, we jointly calibrate: the share of the modelled production sector taxbase that is taxable under the corporate income tax system $\phi_{j,0}^{cit}$, debt to asset ratios $\mu_{j,0}^{c}$; and the proportion of depreciation that can be deducted for corporate income tax purposes $\phi_{j,0}^{k}$, to match each sector's (based on ATO, 2018b) corporate tax revenue as a share of GDP, debt interest

7 Given superannuation is not explicitly modelled in this version of OLGA we group superannuation fund tax together with personal income tax and abstract from the concessional nature of taxation on contributions and earnings.

deductions as a share of GDP, and depreciation deductions as a share of GDP using the following iterative approach.

(i) Calculate an initial estimate of the incorporated share in each sector based on data on incorporated and unincorporated capital stocks and capital rental prices reported in the ABS Estimates of Industry Multifactor Productivity publication (ABS, 2021d).

(ii) Calibrate the debt-to-asset ratio required to match interest expenses reported in company tax return data.

(iii) Calibrate the proportion of depreciation deducted for tax purposes to match those reported in the company tax return data. In practice, firms can only claim depreciation at the historical cost of an asset and because of inflation the full value of depreciation can therefore not be deducted. We estimate that inflation reduces the net present value of depreciation deductions by around 15 per cent, so we limit the proportion of depreciation that can deducted to a maximum value of 0.85.

(iv) If the result of the above procedure does not match the tax revenue, interest expenses and depreciation deductions as reported in the company tax data, then return to step 1. This process is repeated until the model matches the reported data as closely as possible.

Using this process yields the corporate finance and tax parameters summarised in Table 13.

Table 13: Company tax parameters

Parameter				*j* =			
	AGR	MIN	MAN	UTL	CST	SRV	DWE
$\phi^{cit}_{j,0}$	0.22	0.99	0.65	0.35	0.58	0.83	0.00
$\mu^{c}_{j,0}$	0.44	0.43	0.70	0.65	0.60	0.29	0.30
$\phi^{i}_{j,0}$				0			
$\phi^{k}_{j,0}$	0.71	0.85	0.27	0.85	0.29	0.41	0.00
$cit_{j,0}$	0.1%	0.6%	0.3%	0.0%	0.3%	3.0%	0.0%

Source: Authors' calculation based on ATO (2018b) and ABS (2021a, 2021d) data. The unit of $cit_{j,0}$ is % of GDP.

Interest withholding tax

The effective withholding tax rate τ^{wt}_{0} for non-residents is set to 3 per cent. This ensures interest withholding tax revenue as a percentage of GDP matches the estimate in ABS (2021b) of 0.1 per cent. This parameter could be thought of as the weighted average of statutory rates for all countries with which Australia does or does not have a preferential tax treatment.

Payroll tax

In reality, payroll tax rates vary by state and only apply to firms above a certain size. However, because we do not model sub-national economies or differentiate firms by size, we model payroll tax as an industry specific effective rate calculated from data on payroll tax paid and compensation of employees (ABS 2018a and 2020b). The effective rates are reported in Table 14 below for all the sectors.

Table 14: Effective payroll tax rates

Parameter	j = AGR	MIN	MAN	UTL	CST	SRV	DWE
$\tau_{j,0}^{prt}$	0.014	0.044	0.035	0.038	0.022	0.026	0.000

Source: Authors' calculation based on ABS (2018a and 2020b) data.

Goods and services tax

Consistent with current legislation the statutory GST rate τ_0^{gst} is set to 10 per cent. The GST coverage factors $\omega_{j,0}^{c}, \omega_{j,k,0}^{i}, \omega_{j,k,0}^{z}$ are calibrated based on the ABS (2018a) Input-Output tables, as reported in Table 15 below.

Table 15: Goods and services tax coverage factors

Parameter	j = AGR	MIN	MAN	UTL	CST	SRV	DWE
$\omega_{j,0}^{c}$	0.0565	0.8954	0.6829	0.6931	0.8369	0.5523	0.0005
$\omega_{j,AGR,0}^{i}$	0.1202	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\omega_{j,MIN,0}^{i}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\omega_{j,MAN,0}^{i}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\omega_{j,UTL,0}^{i}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\omega_{j,CST,0}^{i}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\omega_{j,SRV,0}^{i}$	0.0000	0.0000	0.1750	0.0000	0.0000	0.1150	0.0000
$\omega_{j,DWE,0}^{i}$	0.0000	0.0000	0.0000	0.0000	1.0000	0.5830	0.0000
$\omega_{j,AGR,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0007	0.0685
$\omega_{j,MIN,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0058	0.3682
$\omega_{j,MAN,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0082	0.8185
$\omega_{j,UTL,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0116	1.0708
$\omega_{j,CST,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0194	0.8764
$\omega_{j,SRV,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0426	0.1208
$\omega_{j,DWE,0}^{z}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Source: Authors' calculation based on ABS (2018a) and ABS (2020a) data.

The GST coverage factors for investment goods by sector have been disaggregated across different purchasing sectors using the same approach as that used to allocate investment goods from different supplying sectors to purchasing sectors.

Other indirect taxes

In a similar vein to the GST coverage factors, effective ad valorem rates for other indirect taxes such as duties and subsidies for various expenditure bundles $\tau_{j,0}^{oit,c}, \tau_{j,k,0}^{oit,i}, \tau_{j,k,0}^{oit,z}$ are calibrated based on the ABS (2018a) Input-Output table, as reported in Table 16 below.

Table 16: Effective indirect tax rates

Parameter				j =			
	AGR	MIN	MAN	UTL	CST	SRV	DWE
$\tau_{j,0}^{oit,c}$	0.0030	0.0051	0.0422	-0.0004	0.0000	0.0118	0.0000
$\tau_{j,AGR,0}^{oit,i}$				0.0016			
$\tau_{j,MIN,0}^{oit,i}$				0.0000			
$\tau_{j,MAN,0}^{oit,i}$				0.0112			
$\tau_{j,UTL,0}^{oit,i}$				0.0000			
$\tau_{j,CST,0}^{oit,i}$				0.0701			
$\tau_{j,SRV,0}^{oit,i}$				0.0125			
$\tau_{j,DWE,0}^{oit,i}$				0.0000			
$\tau_{j,AGR,0}^{oit,z}$	0.0030	0.0002	0.0060	0.0014	0.0017	0.0046	0.0000
$\tau_{j,MIN,0}^{oit,z}$	0.0051	0.0053	0.0152	0.0174	0.0104	0.0241	0.01188
$\tau_{j,MAN,0}^{oit,z}$	0.0422	0.0278	0.0103	0.0413	0.0080	0.0359	0.0059
$\tau_{j,UTL,0}^{oit,z}$	-0.0004	-0.0086	0.0006	0.0003	0.000	-0.0065	0.0005
$\tau_{j,CST,0}^{oit,z}$	0.0000	-0.0014	0.0076	0.0002	0.0000	0.0000	0.0000
$\tau_{j,SRV,0}^{oit,z}$	0.0118	-0.0128	0.0045	0.0061	0.0042	0.0046	0.0110
$\tau_{j,DWE,0}^{oit,z}$	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.0000

Source: Authors' calculation based on ABS (2018a) data.

Spending

The ratio of government spending to GDP in the initial steady state g_0 is the level necessary to ensure a stable government debt to GDP ratio. This is calculated as modelled government revenue less modelled government transfers and the debt stabilising primary balance. The implied government spending to GDP ratio is lower than suggested in the data (ABS, 2021e) because there are several state and local government taxes that are not captured in the model and some government transfers are modelled to be larger share of GDP than reported in the data.

Transfers

Age pension

The age pension function parameters including, age eligibility, maximum pension rate, income threshold, income test taper rate, asset threshold, and asset test taper rate, match actual values as of December 2017 (see Table 17).

Under the current system, an individual's age pension depends on their relationship status, with single payments higher than for those in a couple. We calculate a maximum age pension payment by weighting the rates for single and couple recipients by the proportion of each type of recipients reported by the Department of Social Services (2018). The maximum age pension payment in the model is calibrated to be the same as of December 2017 by adjusting the replacement ratio parameter ϕ^{pen}.

Consistent with the calculation of the maximum age pension payment, the income and asset test thresholds are weighted averages of the single and couple thresholds[8].

Table 17: Age pension parameters

Parameter	Variable	Value
Eligibility age	a	66
Maximum benefit-Replacement ratio	ϕ_0^{pen}	0.26
Maximum age pension payment	pmax	$20,414.94
Asset test-lower bound (max benefit)	$\overline{v}_{1,0}$	$366,379
Asset test-upper bound (no benefit)	$\overline{v}_{2,0}$	$621,566
Asset test-taper rate	ω_0^v	0.08
Income test-lower bound (max benefit)	$\overline{yh_{1,0}}$	$4,110
Income test-upper bound (no benefit)	$\overline{yh_{2,0}}$	$44,940
Income test-taper rate	ω_0^y	0.5
Deemed income-threshold	v_0^{dm}	$50,200
Deemed income-deeming rate below threshold	ω_0^{dm1}	0.0175
Deemed income-deeming rate above threshold	ω_0^{dm2}	0.0325

Other transfers

Other social transfers are calculated from SIH (ABS, 2017). These data indicate that individuals in the bottom income quintile who are close to age pension eligibility generally receive some other transfer payment of similar value. In contrast, the other social transfer payments to the top three income quintiles are generally low and we set them to zero at all ages. In light of this, transfers payments received by the lowest household skill type in the year before they are eligible for age pension

8 Source: https://www.humanservices.gov.au/individuals/enablers/income-test-pensions/30406

eligibility is equal to the maximum age pension payment. Total other social transfers are around 5 per cent of GDP, consistent with personal benefit payments less age pension expenditure reported in government budget outcomes (Commonwealth of Australia, 2016).

Government's net debt

The initial steady state government net debt to GDP ratio is calibrated to be 20 per cent of GDP. This is consistent with the level of the Commonwealth government's net debt before the beginning of the COVID-19 pandemic (Commonwealth of Australia, 2021a).

Financial sector

As noted above the production and hence financial sector calibration in Version 1.0 of OLGA ignores non-produced assets such as land by only including produced assets.

As previously discussed, we assume that firms maintain fixed debt to equity ratios while households hold constant shares of domestic debt and equity assets, and foreign assets. Given these assumptions, the calibration of corporate taxes and finance in the incorporated sector of the economy (described above) determines the overall debt-to-equity ratio in the economy. The resulting aggregate debt-to-equity ratio in the model is 0.57, which is close to the 0.63 implied by the ABS Australian National Accounts: Finance and Wealth publication (ABS, 2021c).

Subject to this estimate, the remaining rates of return, ratios and shares are calibrated to match historical estimates of the following key financial variables:

(i) Consistent with ABS (2021a) the capital stock to GDP ratio is around 3.2. The global benchmark required rate of return is adjusted to target this ratio;

(ii) Consistent with the ABS (2021c) the foreign ownership share of corporate bonds is around 0.85;

(iii) Consistent with ABS (2021c) the foreign ownership share of equity is around 0.37. Additional data are used to calibrate the split between capital gains and dividends, with all earnings accruing to foreign investors are returned as a capital gains;

(iv) Consistent with ATO (2018b), the domestic split between earnings returned as a capital gain or dividend is calibrated to achieve an initial franking credit share of GDP of roughly one per cent;

(v) As noted above, consistent with Commonwealth of Australia (2021a) government debt is calibrated to 20 per cent of GDP;

(vi) Consistent with Australian Office of Financial Management (AOFM (2019)) the foreign ownership share of government debt is around 0.55;

(vii) The discount factor β is adjusted so that the the net foreign liability to GDP ratio is consistent with ABS (2021f). A drawback of this approach is that household foreign investment is modelled as the net rather than gross position. Because the current version of the model ignores the asset value of residential land this approach understates the gross foreign liabilities held abroad by Australian households.

Table 18 shows the ownership of assets in the steady state calibration. In general, these match the financial ratios reported above well.

Table 18: Ownership of assets (per cent of GDP)

Debtor / Creditor	Households	Firms					Government	Total Domestic	Total Foreign	Gross Assets
		Bonds	Equity			Total				
			Dividends	Capital gain	Total					
Households		37 (2)	143 (4)	7 (4)	150 (3)	187	8 (6)	195	102 (7)	298
Firms										
Government										
Foreign		82 (2)		59 (3)	59 (3)	141	12 (6)	153		
Gross Liabilities		119	143	66	209	328 (1)	20 (5)	348		-50* (7)

Source: Authors' calculation. Note:*denotes Australia's net foreign asset position.

Household investment

The domestic and foreign ownership shares in Table 18 imply the share of household savings allocated to non-dwelling assets, which include domestic equity and corporate bonds, government bonds and foreign assets, reported in Table 19.

The share of household savings allocated to dwellings in the initial steady state $\pi^{e}_{dwe,0}$ is estimated to be 0.27. As noted above, this share ensures that domestic households own all equity in the dwellings sector. Because Version 1.0 of OLGA does not include non-produced factors such as land, $\pi^{e}_{dwe,0}v^{h}_{0}$ only captures the value of dwelling structures (produced assets). This assumption will be relaxed in a development module dedicated to modelling the taxation of housing

Table 19: Allocation of household savings in non-dwellings investment

Debtor	Asset type	Portfolio weight	
Firms	Total		0.49
	Bonds	π_0^{bc}	0.17
	Equity Dividend	π_0^{ed}	0.29
	Equity Capital gain	π_0^{ec}	0.03
Government	Bonds	π_0^{bg}	0.04
Foreign portfolio	Total	π_0^{f}	0.47
Total			1.00

Source: Authors' calculation.

Foreign investment

Foreign ownership shares of Australian domestic equity, corporate bonds and government bonds are estimated as a residual. The implied initial portfolio weights are reported in Table 20.

Table 20: Initial allocation of foreign investment

Debtor	Asset type	Portfolio weight	
Firms	Total		0.92
	Bonds	π_0^{bc*}	0.54
	Equity Dividend	π_0^{ed*}	0.00
	Equity Capital gain	π_0^{ec*}	0.39
Government	Bonds	π_0^{bg*}	0.08
Total			1.00

Source: Authors' calculation.

Risk free rate and risk premia

The small open economy framework implies that the domestic interest rate is exogenous and set by the ROW. In Version 1.0 the global required after tax rate of return for sovereign/government debt r_0^{g*} is assumed to be 3.0 per cent.

The before-tax rates of return of all financial assets are linked to global required after-tax rate of return r_0^{g*} with different risk premia applied (see Table 21). These premia are calibrated to match observed relative rates of return on different assets (AMP Capital (2017)).

Table 21: Risk premia and global required return (per cent)

Debtor	Asset type	Premium		Global required rate of return	
Foreign	Risk free bonds			r_t^{g*}	3.00
	Portfolio	ρ_t^{f*}	1.5	r_t^{f*}	4.50
Government	Bonds	ρ_t^g	1.00	r_t^g	4.12*
Firms	Bonds	$\rho_t^g + \rho_t^c$	1.50	r_t^c	4.64*
	Equity	$\rho_t^g + \rho_t^c + \rho_t^e$	2.40	r_t^e	5.4

Note:*include adjustment for withholding tax. Source: Authors' calculation based on AMP Capital (2017) data.

Model benchmark and performance

A summary of the initial steady state macroeconomic variables from the model is reported in Table 22. Overall, these numbers are well in line with the key Australian macroeconomic variables averaged over the 5-year period to 2015-16.

On the income side, the shares of labour and capital income are very close to their actual values. Total capital stock and hours worked line up well with the observed data, suggesting that returns on capital and wages are finely calibrated. The revenue to GDP ratios for major taxes (such as personal income tax, company income tax, and goods and services tax) also generally match the reported data.

On the expenditure side, because the model requires a positive trade balance to stabilise its net foreign liabilities position, we cannot match the negative trade balance in actual data. As such, the model-generated private consumption share of GDP is slightly lower than that observed in Australia. The model also generates a slightly higher investment to GDP ratio. This is a by-product of ignoring fixed factors that do not depreciate and overstating the required investment to cover depreciation.

In terms of government taxation and spending statistics, the model generally matches data well with a few exceptions. First, the model generates higher age pension expenditure to GDP ratio. This is partly because the model has a larger share of the population over 60 than observed in the data. Second, the model misses a fraction of indirect taxes (such as state and local government stamp duty and land tax), and this leads to lower government tax revenue than the actual data. This is partly offset by personal income tax revenue being higher than in the data. As discussed above, the model generates lower government spending to satisfy the government's budget constraint.

Under these macroeconomic settings, the model does a good job in matching the distribution of Australian household income and wealth measured by Gini coefficients.

Table 22: Initial steady state variables

Variable	Data value	Model value
Factor income (% of GDP)		
Labour income	53.5	54.5
Capital income	46.5	45.5
Factor inputs		
Capital stock (% of GDP)	321	328
Hours worked (billion hours)	20.5	20.3
National accounts (% of GDP)		
Consumption	56.3	55.3
Investment	26.7	28.5
Government spending	18	14.5
Exports	20.3	23.4
Imports	21.5	21.5
Net exports	-1.2	1.8
Govt. Revenue (% of GDP)		
Personal income tax	11.3	13.1
Company income tax	4.3	4.4
Goods and services tax	3.4	3.3
Interest withholding tax	0.1	0.1
Payroll tax	1.3	1.4
Other indirect taxes	3.3	2.5
Govt. Payments (% of GDP)		
Franking credits	1.0	1.1
Age pension expenditure	2.5	4.1
Other government benefits	5.3	5.0
Government net debt (% of GDP)	20	20.0
Net foreign liabilities (% of GDP)	54.7	50.3
Net income shares by income quintile		
Low	0.08	0.08
Lower middle	0.13	0.11
Middle	0.17	0.14
Upper middle	0.23	0.22
High	0.40	0.44
Income Gini Coefficient	0.32	0.35
Wealth shares by income quintile		
Low	0.01	0.05
Lower middle	0.05	0.11
Middle	0.11	0.15
Upper middle	0.21	0.21
High	0.62	0.49
Wealth Gini Coefficient	0.60	0.60

Source: Authors' calculation based on ABS (2019b, 2021a, 2021b, 2021c, 2021e, 2021f), ATO (2018b) and Commonwealth of Australia (2016).

Welfare analysis for policy evaluation

As noted at the beginning of this paper, OLGA like other overlapping generations models has several desirable features which means they are well suited to fiscal policy analysis. Households in OLGA are rational and forward-looking. They have explicit objective functions that are consistent with their expectation of the future path of policy and associated macroeconomic aggregates. This allows an explicit comparison of household welfare across alternative fiscal policies. Moreover, households in OLGA are heterogeneous in their age, skill types, income, and wealth. As such, we can move beyond the macroeconomic aggregates and undertake an individual assessment of each household. This allows Treasury to investigate the distributional and intergenerational effect of fiscal policies.

Following Fehr and Kindermann (2018), and Kudrna and Tran (2018), we measure welfare gains and losses for a household associated with a policy change using a dynamic version of the Hicksian Equivalent Variation (HEV) approach. This approach essentially measures the change in lifetime utility (measured in terms of initial consumption prices and wages) that would leave the household indifferent to the change in policy. If the policy change increases their lifetime utility, the HEV will be positive, and the household will be deemed to be better off.

Let s be the age of the household when the policy change is announced at time t, $\left\{\bar{c}^{a,\ell}_{t+a-1}, 1-\bar{N}^{a,\ell}_{t+a-1}\right\}^{95}_{a=s}$ be the household's level of consumption and leisure assuming no policy change (which is typically the steady state path) for their remaining life and $\left\{\tilde{c}^{a,\ell}_{t+a-1}, 1-\tilde{N}^{a,\ell}_{t+a-1}\right\}^{95}_{a=s}$ be the household's level of consumption and leisure under the policy change for their remaining life. Note that this includes households not yet born (that is, $a < 21$). We estimate the HEV via Δ which is the permanent change in the household's consumption and leisure over their remaining lifetime sufficient to yield the same utility as under the policy change:

$$\sum_{a=s}^{95} \beta^{a-s} \Psi^a U((1+\Delta)\bar{c}^{a,\ell}_{t+a-s}, (1+\Delta)\bar{L}^{a,\ell}_{t+a-s}) = \sum_{a=s}^{95} \beta^{a-s} \Psi^a U(\tilde{c}^{a,\ell}_{t+a-s}, \tilde{L}^{a,\ell}_{t+a-s}) \quad (52)$$

If Δ is positive the household is better off under the policy change. Alternatively, if Δ is negative the household is worse off under the policy change. This approach is similar to the 'consumption equivalence' measure adopted by Nishiyama and Reichling (2015).

Conclusion

The overlapping generations model of the Australian economy (OLGA) described in this paper reflects a significant capability in both physical and human capital. This capability is intended to meet the needs of all Treasury's stakeholders today and well into the future. With tools such as OLGA, Treasury's modelling capability is now on par with recognised international fiscal agencies such as the US CBO and JCT.

OLGA is intended to support Treasury's economy-wide counterfactual fiscal policy analysis. It provides a rigorous tool to quantify the general equilibrium effects of fiscal policy proposals. By including the indirect effects of a policy proposal, OLGA provides a comprehensive assessment of the so-called dynamic cost of a policy proposal. It captures the macroeconomic effects of how a proposal is financed. And it helps Treasury better understand who ends up gaining or losing from the policy – that is, how it affects the welfare of different types of households by age, income, and wealth.

The version of OLGA presented in this paper is referred to as Version 1.0. This version of the model essentially encompasses all the functionality of Treasury's previous fiscal models. But it overcomes several of the limitations of earlier models. OLGA Version 1.0 has also been designed so that it can be enhanced through discrete development modules. To deliver on the needs of stakeholders, Treasury will continue to develop the model to meet specific needs.

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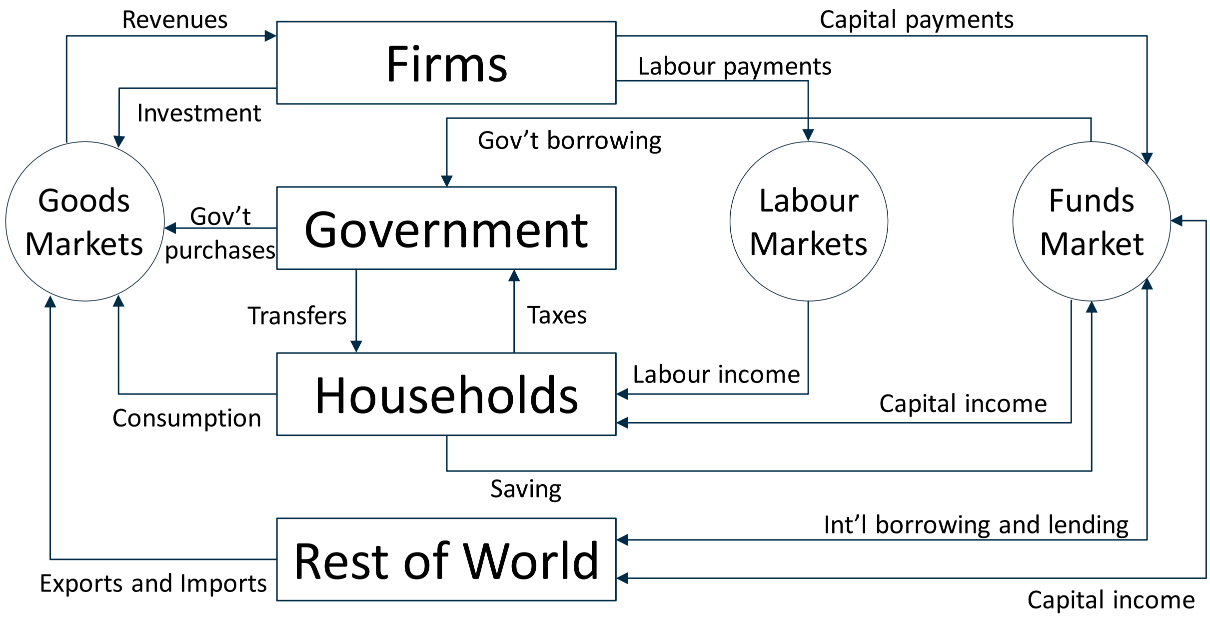
Appendix A: An overview of OLGA

OLGA is a dynamic general equilibrium model with heterogeneous demographic structure and multiple production sectors. There are 75 overlapping generations of households in the model. Households are assumed to be uniformly distributed across five skill types. Households provide labour and savings to the production sectors. Firms in the production sectors employ labour, capital, and intermediate inputs to produce final and intermediate goods and services for domestic use or export. Domestically produced goods compete with differentiated goods and services supplied by foreign producers. Firms manage capital investment decisions, and source funds via a notional funds manager from households and foreign investors. There is a government that collects revenue from households and firms via taxes on production, income, and expenditure. This tax revenue finances government spending. The government relies on funds sourced from domestic and foreign investors to meet temporary primary deficits. The households also own foreign financial assets.

Figure A1:

Overview of the flow of goods/services, factors of production and funds in OLGA



Appendix B: Households' optimisation problem

Bellman equation

As noted in Section 2, a household of skill type ℓ, who enters the model at the beginning of year t, chooses consumption, labour supply and savings to maximise lifetime utility:

$$\sum_{a=21}^{95} \beta^{a-21}\left[\Psi_{t+a-21}^{a} U(c_{t+a-21}^{a,\ell}, L_{t+a-21}^{a,\ell}) + (1-\Psi_{t+a-21}^{a})\Phi(v_{t+a-21}^{a,\ell})\right]$$

where: $0<\beta<1$ is the household's discount factor; $c_{t+a-21}^{a,\ell} \geq 0$, $0 \leq L_{t+a-21}^{a,\ell} \leq 1$, and $v_{t+a-21}^{a,\ell}$ are the household's consumption, leisure, and beginning-of-period savings at year $t+a-21$. This is subject to the intertemporal budget constraint:

$$\begin{aligned}&(1+\gamma_{t+a-21}^{\xi})(v_{t+a+1-21}^{a+1,\ell} - vbq_{t+a+1-21}^{a+1,\ell}) + p_{t+a-21}^{c} c_{t+a-21}^{a,\ell} \\ &= (1+r_{t+a-21}^{h}) v_{t+a-21}^{a,\ell} + w_{t+a-21}{}^{a,\ell} N_{t+a-21}^{a,\ell} - pit_{t+a-21}^{a,\ell} + pen_{t+a-21}^{a,\ell} + tr_{t+a-21}^{a,\ell}\end{aligned}$$

By the principle of optimality (Sundaram 2009, Chapter 11), the household's maximisation problem in period t can be recast with the following Bellman equation:

$$\begin{aligned}V^{t}\left(v_{t}^{a,\ell}\right) &= \max_{N_{t}^{a,\ell}, c_{t}^{a,\ell}, v_{t+1}^{a+1,\ell}} W^{t}(N_{t}^{a,\ell}, c_{t}^{a,\ell}, v_{t+1}^{a+1,\ell}, v_{t}^{a,\ell}) \\ &= \max_{N_{t}^{a,\ell}, c_{t}^{a,\ell}, v_{t+1}^{a+1,\ell}} \left\{U(c_{t}^{a,\ell}, 1-N_{t}^{a,\ell}) + \beta\left(\left(1-\psi_{t}^{a}\right)\Phi\left(v_{t+1}^{a+1,\ell}\right) + \psi_{t}^{a} V^{t+1}\left(v_{t+1}^{a+1,\ell}\right)\right)\right\}\end{aligned} \tag{53}$$

with the intertemporal budget constraint:

$$(1+\gamma_{t}^{\xi})(v_{t+1}^{a+1,\ell} - vbq_{t+1}^{a+1,\ell}) + p_{t}^{c} c_{t}^{a,\ell} = (1+r_{t}^{h}) v_{t}^{a,\ell} + w_{t}^{a,\ell} N_{t}^{a,\ell} - pit_{t}^{a,\ell} + pen_{t}^{a,\ell} + tr_{t}^{a,\ell} \tag{54}$$

Here V^{t} is called the value function of the household's maximisation problem, and W^{t} is the reward function for the household's choice of $N_{t}^{a,\ell}, c_{t}^{a,\ell}, v_{t+1}^{a+1,\ell}$ given initial savings of $v_{t}^{a,\ell}$. Intuitively, $V^{t}\left(v_{t}^{a,\ell}\right)$ is the optimal level of lifetime utility the household can obtain given initial savings $v_{t}^{a,\ell}$ in period t; and $V^{t+1}\left(v_{t+1}^{a+1,\ell}\right)$ is the continuation value for the household with initial savings $v_{t+1}^{a+1,\ell}$ in period $t+1$. This alternative representation using Bellman equation demonstrates the household's recursive problem over time.

Solution technique

Because the household has a finite lifetime, we have:

$$\psi_{t+1}^{96} = 0$$

$$V^{t+1}\left(v_{t+1}^{96,\ell}\right) = 0$$

$$\frac{\partial V^{t+1}\left(v_{t+1}^{96,\ell}\right)}{\partial v_{t+1}^{76,\ell}} = 0$$

We can use this information to solve the household's maximisation problem by backward induction. Specifically, we have the Bellman equation for a household who is 95 years old in period t:

$$V^{t}\left(v_{t}^{95,\ell}\right) = \max_{N_t^{95,\ell}, c_t^{95,\ell}, v_{t+1}^{96,\ell}} W^{t}(N_t^{95,\ell}, c_t^{95,\ell}, v_{t+1}^{96,\ell}; v_t^{95,\ell})$$

$$= \max_{N_t^{95,\ell}, c_t^{95,\ell}, v_{t+1}^{96,\ell}} \left\{U(c_t^{95,\ell}, 1 - N_t^{95,\ell}) + \beta\Phi\left(v_{t+1}^{96,\ell}\right)\right\}$$

Given the functional forms of U, Φ, the reward function W^t is concave in $c_t^{95,\ell}, v_{t+1}^{96,\ell}$. Nevertheless, due to progressivity in personal income tax, means-tested pension and other discontinuous policies, there is no guarantee that W^t is concave in labour supply $N_t^{95,\ell}$. Therefore, we can only use the Karush–Kuhn–Tucker conditions to find the optimal solution for $c_t^{95,\ell}, v_{t+1}^{96,\ell}$, but rely on grid search for $N_t^{95,\ell}$.

We proceed by discretising the household's labour supply. Conditioned on $N_t^{95,\ell} \in [N_1, N_2, ..., N_{\max}]$ for some $N_{\max} < 1$, the optimal level of consumption $c_t^{95,\ell}$ and savings $v_{t+1}^{96,\ell}$ can be solved through the following Lagrangian:

$$\begin{aligned}
L_t = & \max_{c_t^{95,\ell}, v_{t+1}^{96,\ell}} U\left(c_t^{95,\ell}, 1 - N_t^{95,\ell}\right) + \beta\Phi\left(v_{t+1}^{96,\ell}\right) \\
& + \Lambda_t^{95,\ell}\left(\left(1+\gamma_t^{\xi}\right)v_{t+1}^{96,\ell} + p_t^c c_t^{95,\ell} - (1+r_t^h)v_t^{95,\ell} - w_t^{95,\ell}N_t^{95,\ell} + pit_t^{95,\ell} - pen_t^{95,\ell} - tr_t^{95,\ell}\right) \\
& - \Upsilon_t^{95,\ell}\left(\underline{v} - v_{t+1}^{96,\ell}\right)
\end{aligned}$$

The Karush–Kuhn–Tucker conditions imply:

$$c_t^{95,\ell} : \frac{\partial U\left(c_t^{95,\ell}, 1 - N_t^{95,\ell}\right)}{\partial c_t^{95,\ell}} = p_t^c \Lambda_t^{95,\ell} \tag{55}$$

$$v_{t+1}^{96,\ell} : \beta \frac{\partial \Phi\left(v_{t+1}^{96,\ell}\right)}{\partial v_{t+1}^{96,\ell}} - \left(1 + \gamma_t^{\xi}\right) \Lambda_t^{95,\ell} + \Upsilon_t^{95,\ell} = 0 \tag{56}$$

Together with the complementary slackness conditions:

$$\Upsilon_t^{95,\ell} \geq 0 \tag{57}$$

$$\underline{v} - v_{t+1}^{96,\ell} \leq 0 \tag{58}$$

$$\Upsilon_t^{95,\ell}\left(\underline{v} - v_{t+1}^{96,\ell}\right) = 0 \tag{59}$$

By substitution, we have:

$$\beta \frac{\partial \Phi\left(v_{t+1}^{96,\ell}\right)}{\partial v_{t+1}^{96,\ell}} - \frac{\left(1 + \gamma_t^{\xi}\right)}{p_t^c} \frac{\partial U\left(c_t^{95,\ell}, 1 - N_t^{95,\ell}\right)}{\partial c_t^{95,\ell}} + \Upsilon_t^{95,\ell} = 0 \tag{60}$$

The complementarity slackness conditions suggest that either $\Upsilon_t^{95,\ell} = 0$ or $v_{t+1}^{96,j} = \underline{v}$. Therefore, we can proceed by trial-and-error. First, we assume $\Upsilon_t^{95,\ell} = 0$, and solve for $v_{t+1}^{96,\ell}$ that satisfies equation (60). We then check if $v_{t+1}^{96,\ell} > \underline{v}$. If so, we obtain the solution $\tilde{v}_{t+1}^{96,\ell}$; otherwise, we let $\tilde{v}_{t+1}^{96,\ell} = \underline{v}$ as the complementarity slackness condition suggests. Given $v_t^{95,\ell}, N_t^{95,\ell}, \tilde{v}_{t+1}^{96,\ell}$, we can then use the household's intertemporal budget constraint to obtain $\tilde{c}_t^{95,\ell}$.

Proceeding as prescribed above along the grid of labour supply $\left[N_1, N_2, ..., N_{\max}\right]$, we obtain a set of candidate solutions $\left\{N_t^{95,\ell}, \tilde{c}_t^{95,\ell}, \tilde{v}_{t+1}^{96,\ell}\right\}_{1,} \left\{N_t^{95,\ell}, \tilde{c}_t^{95,\ell}, \tilde{v}_{t+1}^{96,\ell}\right\}_{2,} ... \left\{N_t^{95,\ell}, \tilde{c}_t^{95,\ell}, \tilde{v}_{t+1}^{96,\ell}\right\}_{\max}$ to the Bellman equation given initial savings $v_t^{95,\ell}$. The optimal solution $N_t^{95,\ell,*}, c_t^{95,\ell,*}, v_{t+1}^{96,\ell,*}$ is one that maximises the value of the Bellman equation.

Now knowing $N_t^{95,\ell,*}, c_t^{95,\ell,*}, v_{t+1}^{96,\ell,*}$ and $V^t\left(v_t^{95,\ell}\right)$ given initial savings $v_t^{95,\ell}$, we can move backward to solve the household's Bellman equation at age 94 in period $t-1$:

$$\begin{aligned} V^{t-1}\left(v_{t-1}^{94,\ell}\right) &= \max_{N_{t-1}^{94,\ell}, c_{t-1}^{94,\ell}, v_t^{95,\ell}} W^{t-1}(N_{t-1}^{94,\ell}, c_{t-1}^{94,\ell}, v_t^{95,\ell}; v_{t-1}^{94,\ell}) \\ &= \max_{N_{t-1}^{94,\ell}, c_{t-1}^{94,\ell}, v_t^{95,\ell}} \left\{U(c_{t-1}^{94,\ell}, 1 - N_{t-1}^{94,\ell}) + \beta\left(\left(1 - \psi_{t-1}^{94}\right)\Phi\left(v_t^{95,\ell}\right) + \psi_{t-1}^{94} V^t\left(v_t^{95,\ell}\right)\right)\right\} \end{aligned}$$

Following Stachurski (2011, Chapter 5), one can prove that the reward function W^{1-t} is concave in $c_{t-1}^{94,\ell}, v_t^{95,\ell}$. But again, there is no guarantee that W^{t-1} is concave in labour supply $N_{t-1}^{94,\ell}$.

Conditioned on $N_{t-1}^{94,\ell} \in \left[N_1, N_2, ..., N_{\max}\right]$, we have the following Lagrangian:

$$
\begin{aligned}
L_{t-1} = & \max_{c_{t-1}^{94,\ell}, v_{t-1}^{95,\ell}} \left\{ u\left(c_{t-1}^{94,\ell}, 1 - N_{t-1}^{94,\ell}\right) + \beta\left(\left(1-\psi_{t-1}^{94}\right)\Phi\left(v_t^{95,\ell}\right) + \psi_{t-1}^{94} V^t\left(v_t^{95,\ell}\right)\right)\right\} \\
& + \Lambda_{t-1}^{94,\ell}\left(\left(1+\gamma_{t-1}^{\xi}\right)\left(v_t^{95,\ell} - vbq_t^{95,\ell}\right) + p_{t-1}^c c_{t-1}^{94,\ell} - (1+r_{t-1}^h) v_{t-1}^{94,\ell} - w_{t-1}^{94,\ell} N_{t-1}^{94,\ell} + pit_{t-1}^{94,\ell} - pen_{t-1}^{94,\ell} - tr_{t-1}^{94,\ell}\right) \\
& - \Upsilon_{t-1}^{94,\ell}\left(\underline{v} - v_t^{95,\ell}\right)
\end{aligned}
$$

The Karush–Kuhn–Tucker conditions for optimality again imply:

$$c_{t-1}^{94,\ell} : \frac{\partial U\left(c_{t-1}^{94,\ell}, 1 - N_{t-1}^{94,\ell}\right)}{\partial c_{t-1}^{94,\ell}} = p_{t-1}^c \Lambda_{t-1}^{94,\ell} \tag{61}$$

$$v_t^{95,\ell} : \beta\left(\left(1-\psi_{t-1}^{94}\right)\frac{\partial \Phi\left(v_t^{95,\ell}\right)}{\partial v_t^{95,\ell}} + \psi_{t-1}^{94}\frac{\partial V^t\left(v_t^{95,\ell}\right)}{\partial v_t^{95,\ell}}\right) - \left(1+\gamma_{t-1}^{\xi}\right)\Lambda_{t-1}^{94,\ell} + \Upsilon_{t-1}^{94,\ell} = 0 \tag{62}$$

with the complementary slackness conditions:

$$\Upsilon_{t-1}^{94,\ell} \geq 0 \tag{63}$$

$$\underline{v} - v_t^{95,\ell} \leq 0 \tag{64}$$

$$\Upsilon_{t-1}^{94,\ell}\left(\underline{v} - v_t^{95,\ell}\right) = 0 \tag{65}$$

The envelop theorem also suggests:

$$
\begin{aligned}
\frac{\partial V^t\left(v_t^{95,\ell}\right)}{\partial v_t^{95,\ell}} = \frac{\partial L_t}{\partial v_t^{95,\ell}} &= \left(1 + r_t^h + \frac{\partial pen_t^{95,\ell}}{\partial v_t^{95,\ell}} - \frac{\partial pit_t^{95,\ell}}{\partial v_t^{95,\ell}}\right)\Lambda_t^{a,\ell,*} \\
&= \frac{\left(1 + r_t^h + \frac{\partial pen_t^{95,\ell}}{\partial v_t^{95,\ell}} - \frac{\partial pit_t^{95,\ell}}{\partial v_t^{95,\ell}}\right)}{p_t^c}\frac{\partial U\left(c_t^{95,\ell,*}, 1 - N_t^{95,\ell,*}\right)}{\partial c_t^{95,\ell,*}}
\end{aligned} \tag{66}
$$

By substitution, we have:

$$\beta\left(\left(1-\psi_{t-1}^{94}\right)\frac{\partial\Phi\left(v_t^{95,\ell}\right)}{\partial v_t^{95,\ell}}+\psi_{t-1}^{94}\frac{\left(1+r_t^h+\frac{\partial pen_t^{95,\ell}}{\partial v_t^{95,\ell}}-\frac{\partial pit_t^{95,\ell}}{\partial v_t^{95,\ell}}\right)}{p_t^c}\frac{\partial U\left(c_t^{95,\ell,*},1-N_t^{95,\ell,*}\right)}{\partial c_t^{95,\ell,*}}\right)-\frac{\left(1+\gamma_{t-1}^{\xi}\right)}{p_{t-1}^c}\Lambda_{t-1}^{94,\ell}+\Upsilon_{t-1}^{94,\ell}=0$$

We solve this equation with the intertemporal budget constraint and complementarity slackness conditions in a similar way as discussed above to obtain the optimal solutions to the household's Bellman equation at age 94 in period $t-1$. After this, we go further backward and repeat the process until we derive the optimal solutions $N_t^{a,\ell,*}, c_t^{a,\ell,*}, v_{t+1}^{a,\ell,*}$ for any initial savings $v_t^{a,\ell}$ from age 21 to 95.

Knowing the optimal decision rules, we can finally work forward to derive the household's optimal life-time labour supply, consumption, and savings, given initial savings at age 21.

Appendix C: Firm's optimisation problem

As noted in Section 2, the objective for the representative firm in each sector is to choose the level of production (that is, the level of capital, labour and intermediate inputs) to maximise the market value of equity:

$$v_{j,t}^{c} = \sum_{s=t}^{\infty} \left(\prod_{u=t}^{s} \frac{1+\gamma_u}{1+r_u^e} \right) \left(\frac{q_{j,s}}{1+\gamma_s} \right)$$

This is subject to the debt-to-capital ratio:

$$b_{j,t}^{c} = \mu_{j,t}^{b} p_{j,t}^{k} k_{j,t}$$

earnings before interest, tax and amortisation:

$$e_{j,t} = p_{j,t}^{y} y_{j,t} - (1+\tau_t^{prt}) w_t \tilde{n}_{j,t} - p_{j,t}^{z} z_{j,t}$$

company income tax:

$$cit_{j,t} = \tau_{j,t}^{cit} (e_{j,t} - r_t^c b_{j,t}^c - \phi_{j,t}^k \delta_{j,t} p_{j,t}^i k_{j,t} - \phi_{j,t}^i p_{j,t}^i i_{j,t})$$

budget constraint:

$$(1+r_t^c) b_{j,t}^c + cit_{j,t} + q_{j,t} + p_{j,t}^i i_{j,t} = e_{j,t} + (1+\gamma_t) b_{j,t+1}^c$$

law of motion for capital:

$$(1+\gamma_t) k_{j,t+1} = (1-\delta_{j,t}) k_{j,t} + i_{j,t}$$

and production technology:

$$y_{j,t} = \lambda_{j,t} \left[\theta_j^{yn} (\tilde{n}_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}} + \theta_j^{yk} (k_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}} + \theta_j^{yz} (z_{j,t})^{\frac{\eta_j^y - 1}{\eta_j^y}} \right]^{\frac{\eta_j^y}{\eta_j^y - 1}} - \frac{\zeta_j^k}{2} \left(\frac{i_{j,t}}{k_{j,t}} - \delta_{j,t} - \gamma_t \right)^2 k_{j,t}$$

First order conditions

Dropping the subscript for a production sector, the Lagrangian for the representative firm's problem is:

$$\mathcal{L} = \sum_{s=t}^{\infty}\left(\prod_{u=t}^{s}\frac{1+\gamma_u}{1+r_u^e}\frac{1}{1+\gamma_s}\right)\left\{\begin{array}{l} q_s \\ +\Lambda_s^b\left(\mu_s^b p_s^k k_s - b_s^c\right) \\ +\Lambda_s^e\left(p_s^y y_s - (1+\tau_s^{prt})w_s\tilde{n}_s - p_s^z z_s - e_s\right) \\ +\Lambda_s^{cit}\left(\tau_s^{cit}\left(e_s - r_s^c b_s^c - \varphi_s^k\delta_s p_s^i k_s - \varphi_s^i p_s^i i_s\right) - cit_s\right) \\ +\Lambda_s^q\left(e_s + (1+\gamma_s)b_{s+1}^c - \left(1+r_s^c\right)b_s^c - cit_s - p_s^i i_s - q_s\right) \\ +\Lambda_s^k\left((1-\delta_s)k_s + i_s - (1+\gamma_s)k_{s+1}\right) \\ +\Lambda_s^y\left(\begin{array}{l}\lambda_s\left[\left(\theta^{yn}\right)^{\frac{1}{\eta^y}}\left(\tilde{n}_s\right)^{\frac{\eta^y-1}{\eta^y}} + \left(\theta^{yk}\right)^{\frac{1}{\eta^y}}\left(k_s\right)^{\frac{\eta^y-1}{\eta^y}} + \left(\theta^{yz}\right)^{\frac{1}{\eta^y}}\left(z_s\right)^{\frac{\eta^y-1}{\eta^y}}\right]^{\frac{\eta^y}{\eta^y-1}} \\ -\frac{\zeta^k}{2}\left(\frac{i_s}{k_s} - \delta_s - \gamma_s\right)^2 k_s - y_s\end{array}\right)\end{array}\right\}$$

Here Λ_t^b, Λ_t^e, Λ_t^{cit}, Λ_t^q, Λ_t^k and Λ_t^y denote the shadow prices of the constraints. The first order conditions of the above problem therefore imply:

$$\Lambda_t^q = 1 \tag{67}$$

$$\Lambda_t^{cit} = -\Lambda_t^q = -1 \tag{68}$$

$$\Lambda_t^e = \Lambda_t^q + \Lambda_t^{cit}\tau_t^{cit} = \left(1-\tau_t^{cit}\right) \tag{69}$$

$$\Lambda_t^y = \Lambda_t^e p_t^y = \left(1-\tau_t^{cit}\right)p_t^y \tag{70}$$

$$\Lambda_t^k = p_t^i + \left(1-\tau_t^{cit}\right)p_t^y\zeta^k\left(\frac{i_t}{k_t} - \delta_t - \gamma_t\right) - \tau_t^{cit}\varphi_t^i p_t^i \tag{71}$$

$$(1+\tau_t^{prt})w_t\Lambda_t^e = \Lambda_t^y\lambda_t\left(\theta^{yn}\right)^{\frac{1}{\eta^y}}\left(\tilde{n}_t\right)^{-\frac{1}{\eta^y}}\left(y_t/\lambda_t\right)^{\frac{1}{\eta^y}}$$
$$=\left(1-\tau_t^{cit}\right)p_t^y\lambda_t\left(\theta^{yn}\right)^{\frac{1}{\eta^y}}\left(\tilde{n}_t\right)^{-\frac{1}{\eta^y}}\left(y_t/\lambda_t\right)^{\frac{1}{\eta^y}} \tag{72}$$

$$p_t^z\cdot\Lambda_t^e = \Lambda_t^y\lambda_t\left(\theta^{yz}\right)^{\frac{1}{\eta^y}}\left(z_t\right)^{-\frac{1}{\eta^y}}\left(y_t/\lambda_t\right)^{\frac{1}{\eta^y}} = \left(1-\tau_t^{cit}\right)p_t^y\lambda_t\left(\theta^{yz}\right)^{\frac{1}{\eta^y}}\left(z_t\right)^{-\frac{1}{\eta^y}}\left(y_t/\lambda_t\right)^{\frac{1}{\eta^y}} \tag{73}$$

$$\Lambda_t^q = \frac{\Lambda_{t+1}^b+\left(1+r_{t+1}^c\right)-\tau_{t+1}^{cit}r_{t+1}^c}{\left(1+r_{t+1}^e\right)} \tag{74}$$

or equivalently:

$$\Lambda_{t+1}^b = r_{t+1}^e - r_{t+1}^c + \tau_{t+1}^{cit}r_{t+1}^c \tag{75}$$

And we finally have:

$$\left(1+r_{t+1}^e\right)\cdot\Lambda_t^k$$
$$=\Lambda_{t+1}^b\mu_{t+1}^b p_{t+1}^k - \Lambda_{t+1}^{cit}\tau_{t+1}^{cit}\varphi_{t+1}^k\delta_{t+1}p_{t+1}^i$$
$$+\Lambda_{t+1}^k\left(1-\delta_{t+1}\right)+\Lambda_{t+1}^e p_{t+1}^y\left[-\frac{\zeta^k}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^2+\zeta^k\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)\frac{i_{t+1}}{k_{t+1}}\right] \tag{76}$$
$$+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^y\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^y}}\left(k_{t+1}\right)^{-\frac{1}{\eta^y}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^y}}$$

This is the no-arbitrage condition for investment which links current capital shadow price Λ_t^k to its future counterpart Λ_{t+1}^k.

Market value of equity

We claim that if

$$p_{t+1}^k = \Lambda_t^k \tag{77}$$

Then

$$v_{t+1}^c = \left(1-\mu_{t+1}^b\right)\Lambda_t^k k_{t+1} \tag{78}$$

To see this, following Obstfeld and Rogoff (1996, Chapter 2), we multiply both sides of equation (76) by k_{t+1} and substitute for Λ_{t+1}^{b} and Λ_{t+1}^{cit}, which gives:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\Lambda_{t}^{k}k_{t+1} \\
&=\Lambda_{t+1}^{b}\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&-\Lambda_{t+1}^{cit}\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\Lambda_{t+1}^{k}\left(1-\delta_{t+1}\right)k_{t+1}+\Lambda_{t+1}^{e}p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1})i_{t+1}\right] \\
&+q_{t+1}k_{t+1} \\
&=\left(r_{t+1}^{e}-r_{t+1}^{c}+\tau_{t+1}^{cit}r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\Lambda_{t+1}^{k}\left(1-\delta_{t+1}\right)k_{t+1}+\Lambda_{t+1}^{e}p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1})i_{t+1}\right] \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned}
\tag{79}
$$

Subtracting $\left(1+r_{t+1}^{e}\right)\Lambda_{t}^{k}\mu_{t+1}^{b}k_{t+1}$ from both sides of the equation above and substituting for Λ_{t}^{k} on the right-hand side, we further have:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=-\left(1+r_{t+1}^{c}-\tau_{t+1}^{cit}r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta\, p_{t+1}^{i}k_{t+1} \\
&+\Lambda_{t+1}^{k}\left(1-\delta_{t+1}\right)k_{t+1}+\Lambda_{t+1}^{e}p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1})i_{t+1}\right] \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned}
$$

Now using the law of motion for capital as per equation (11), we have:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=-\left(1+r_{t+1}^{c}-\tau_{t+1}^{cit}r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\Lambda_{t+1}^{k}\left((1+\gamma_{t+1})k_{t+2}-i_{t+1}\right)+\Lambda_{t+1}^{e}p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)i_{t+1}\right] \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned}
$$

Substituting for Λ_{t}^{e}, we have:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=-\left(1+r_{t+1}^{c}-\tau_{t+1}^{cit}r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\Lambda_{t+1}^{k}\left((1+\gamma_{t+1})k_{t+2}-i_{t+1}\right) \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)i_{t+1}\right] \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned}
$$

Substituting for Λ_{t}^{k}, we have:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=-\left(1+r_{t+1}^{c}-\tau_{t+1}^{cit}r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1}+\mu_{t+2}^{b}\Lambda_{t+1}^{k}(1+\gamma_{t+1})k_{t+2} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&-\left(p_{t+1}^{i}+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\zeta^{k}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)-\tau_{t+1}^{cit}\varphi_{t+1}^{i}p_{t+1}^{i}\right)i_{t+1} \\
&+\left(1-\mu_{t+2}^{b}\right)\Lambda_{t+1}^{k}(1+\gamma_{t+1})k_{t+2} \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\left[-\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1}+\zeta^{k}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)i_{t+1}\right] \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned}
$$

Further algebra will then give:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=-\left(1+r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1}+\mu_{t+2}^{b}\Lambda_{t+1}^{k}(1+\gamma_{t+1})k_{t+2} \\
&-p_{t+1}^{i}i_{t+1} \\
&+\tau_{t+1}^{cit}r_{t+1}^{c}\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{i}p_{t+1}^{i}i_{t+1} \\
&+(1+\gamma_{t+1})\left(1-\mu_{t+2}^{b}\right)\Lambda_{t+1}^{k}k_{t+2} \\
&-\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1} \\
&+\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1}
\end{aligned} \tag{80}
$$

Because the production is homogenous of degree one, by Euler's theorem and the first order conditions for labour and intermediate inputs, we have:

$$
\begin{aligned}
&p_{t+1}^{y}\lambda_{t+1}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{t+1}\right)^{-\frac{1}{\eta^{y}}}\left(y_{t+1}/\lambda_{t+1}\right)^{\frac{1}{\eta^{y}}}k_{t+1} \\
&=p_{t+1}^{y}y_{t+1}-\frac{\partial\Gamma}{\partial\tilde{n}_{t+1}}\tilde{n}_{t+1}-\frac{\partial\Gamma}{\partial z_{t+1}}z_{t+1} \\
&=p_{t+1}^{y}y_{t+1}-(1+\tau_{t+1}^{prt})w_{t+1}\tilde{n}_{t+1}-p_{t+1}^{z}z_{t+1}
\end{aligned} \tag{81}
$$

This implies:

$$
\begin{aligned}
&\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1} \\
&=(1+\gamma_{t+1})\left(1-\mu_{t+2}^{b}\right)\Lambda_{t+1}^{k}k_{t+2} \\
&-\left(1-\tau_{t+1}^{cit}\right)p_{t+1}^{y}\frac{\zeta^{k}}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\delta_{t+1}-\gamma_{t+1}\right)^{2}k_{t+1} \\
&+\left(1-\tau_{t+1}^{cit}\right)\left(p_{t+1}^{y}y_{t+1}-(1+\tau_{t+1}^{prt})w_{t+1}\tilde{n}_{t+1}-p_{t+1}^{z}z_{t+1}\right) \\
&+\tau_{t+1}^{cit}r_{t+1}^{c}\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{k}\delta_{t+1}p_{t+1}^{i}k_{t+1} \\
&+\tau_{t+1}^{cit}\varphi_{t+1}^{i}p_{t+1}^{i}i_{t+1} \\
&-p_{t+1}^{i}i_{t+1} \\
&-\left(1+r_{t+1}^{c}\right)\mu_{t+1}^{b}p_{t+1}^{k}k_{t+1}+\mu_{t+2}^{b}\Lambda_{t+1}^{k}(1+\gamma_{t+1})k_{t+2}
\end{aligned} \tag{82}
$$

Using the firm's budget constraint, the equation above can be further rewritten as:

$$\left(1+r_{t+1}^{e}\right)\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1}=(1+\gamma_{t+1})\left(1-\mu_{t+2}^{b}\right)\Lambda_{t+1}^{k}k_{t+2}+q_{t+1} \quad (83)$$

By forward iteration, we have replicated equation (18) as follows:

$$\left(1-\mu_{t+1}^{b}\right)\Lambda_{t}^{k}k_{t+1}=\sum_{s=t+1}^{\infty}\left(\prod_{u=t+1}^{s}\frac{1+\gamma_{u}}{1+r_{u}^{e}}\right)\frac{q_{s}}{1+\gamma_{s}}=v_{t+1}^{c}$$

Steady state conditions

At the steady state, we have:

$$\Lambda_{\infty}^{k}=\left(1-\tau_{\infty}^{cit}\varphi_{\infty}^{i}\right)p_{\infty}^{i}$$

$$\Lambda_{\infty}^{b}=r_{\infty}^{e}-\left(1-\tau_{\infty}^{cit}\right)r_{\infty}^{c}$$

$$\left(1-\tau_{\infty}^{cit}\right)p_{\infty}^{y}\lambda_{\infty}\left(\theta^{yk}\right)^{\frac{1}{\eta^{y}}}\left(k_{\infty}\right)^{-\frac{1}{\eta^{y}}}\left(y_{\infty}/\lambda_{\infty}\right)^{\frac{1}{\eta^{y}}}=\left(1-\tau_{\infty}^{cit}\varphi_{\infty}^{i}\right)\left(r_{\infty}^{e}+\delta_{\infty}\right)p_{\infty}^{i}-\Lambda_{\infty}^{b}\mu_{\infty}^{b}p_{\infty}^{k}-\tau_{\infty}^{cit}\varphi_{\infty}^{k}\delta_{\infty}p_{\infty}^{i}$$

Manipulating the last equation above gives the following:

$$p_{\infty}^{y}\lambda_{\infty}\left(\theta^{yk}\right)^{\frac{1}{\eta^{yk}}}\left(k_{\infty}\right)^{-\frac{1}{\eta^{y}}}\left(y_{\infty}/\lambda_{\infty}\right)^{\frac{1}{\eta^{y}}}$$
$$=\frac{r_{\infty}^{e}\left(p_{\infty}^{i}-\mu_{\infty}^{b}p_{\infty}^{k}\right)+\left(1-\tau_{\infty}^{cit}\right)r_{\infty}^{c}\mu_{\infty}^{b}p_{\infty}^{k}-\tau_{\infty}^{cit}\varphi_{\infty}^{i}r_{\infty}^{e}p_{\infty}^{i}-\tau_{\infty}^{cit}\varphi_{\infty}^{i}\delta_{\infty}p_{\infty}^{i}-\tau_{\infty}^{cit}\varphi_{\infty}^{k}\delta_{\infty}p_{\infty}^{i}+\delta_{\infty}p_{\infty}^{i}}{1-\tau_{\infty}^{CIT}}$$

Letting $\overline{rp}_{\infty}$ be firm's effective cost of capital net of tax deduction:

$$\overline{rp}_{\infty}=r_{\infty}^{e}\left(p_{\infty}^{i}-\mu_{\infty}^{b}p_{\infty}^{k}\right)+\left(1-\tau_{\infty}^{cit}\right)r_{\infty}^{c}\mu_{\infty}^{b}p_{\infty}^{k}-\tau_{\infty}^{cit}\varphi_{\infty}^{i}r_{\infty}^{e}p_{\infty}^{i}-\tau_{\infty}^{cit}\varphi_{\infty}^{i}\delta_{\infty}p_{\infty}^{i}-\tau_{\infty}^{cit}\varphi_{\infty}^{k}\delta_{\infty}p_{\infty}^{i}$$

We have:

$$p_{\infty}^{y}\frac{\partial y_{\infty}}{\partial k_{\infty}}=\frac{\overline{rp}+\delta_{\infty}p_{\infty}^{i}}{1-\tau_{\infty}^{cit}}$$

This implies that at the steady state, the marginal return on capital after company income tax is equal to the effective cost of capital plus depreciation.

Using the first second conditions, we also have:

$$k_\infty = \theta^{yk} \frac{y_\infty}{\lambda_\infty} \left(\frac{\frac{\overline{rp} + \delta_\infty p_\infty^i}{1-\tau_\infty^{cit}}}{p_\infty^y \lambda_\infty} \right)^{-\eta^y}$$

$$\tilde{n}_\infty = \theta^{yn} \frac{y_\infty}{\lambda_\infty} \left(\frac{w_\infty}{p_\infty^y \lambda_\infty} \right)^{-\eta^y}$$

$$z_\infty = \theta^{yz} \frac{y_\infty}{\lambda_\infty} \left(\frac{p_\infty^z}{p_\infty^y \lambda_\infty} \right)^{-\eta^y}$$

These equations suggest the following relationship between the CES weights and other variables:

$$\theta^{yk} = \frac{k_\infty \frac{\overline{rp} + \delta_\infty p_\infty^i}{1-\tau_\infty^{cit}}}{y_\infty p_\infty^y} \left(\frac{\frac{\overline{rp} + \delta_\infty p_\infty^i}{1-\tau_\infty^{cit}}}{p_\infty^y \lambda_\infty} \right)^{-\eta^y} \quad (84)$$

$$\theta^{yn} = \frac{\tilde{n}_\infty w_\infty}{y_\infty p_\infty^y} \left(\frac{w_\infty}{p_\infty^y \lambda_\infty} \right)^{-\eta^y} \quad (85)$$

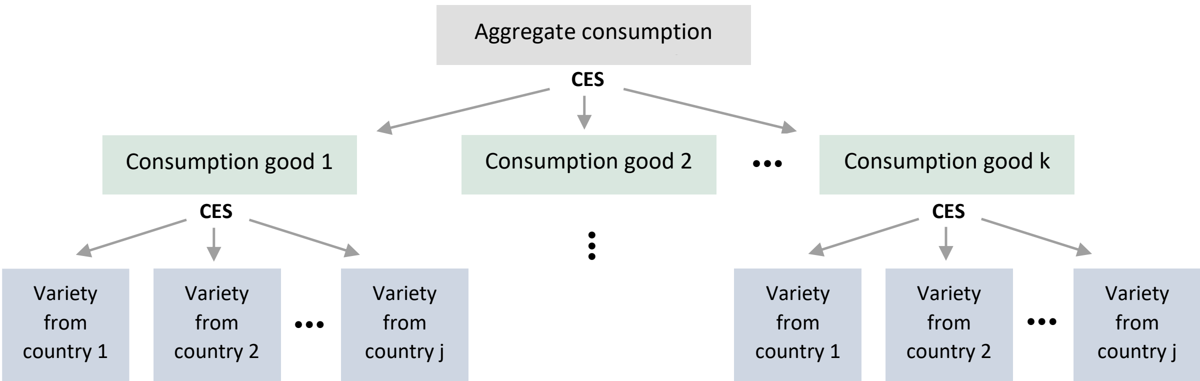
$$\theta^{yz} = \frac{z_\infty p_\infty^y}{y_\infty p_\infty^y} \left(\frac{p_\infty^y}{p_\infty^y \lambda_\infty} \right)^{-\eta^y} \quad (86)$$

It is worth noting that, $\frac{k_\infty \frac{\overline{rp} + \delta_\infty p_\infty^i}{1-\tau_\infty^{cit}}}{y_\infty p_\infty^y}$, $\frac{\tilde{n}_\infty w_\infty}{y_\infty p_\infty^y}$ and $\frac{z_\infty p_\infty^y}{y_\infty p_\infty^y}$ above are the cost shares of capital, labour and intermediate inputs. Assuming that the Australian economy is at steady state, we can then use equations (84) to (86) and the cost share information from Table 4 to estimate the CES weights.

Appendix D: Modelling foreign import demand (aka Demand for Australian exports)

Consider a representative consumer living in foreign country who solves a nested utility maximising problem. At the top of the nest is the choice between different types of consumption goods (for example iron ore, computers, and foods). Following this is the choice between different varieties of the same consumption good that are produced in different exporting countries (for example Australian iron ore and Brazilian iron ore).

Figure A2: Foreign consumer's nested preference



Assuming the foreign consumer’s preferences at the top tier are captured by a constant elasticity of substitution utility function (CES) preference, the problem can be summarised by the following:

$$\max_{c_j^*} c^* = \left[\sum_j \left(\theta_j^{c*}\right)^{\frac{1}{\eta^{c*}}} c_j^{*\,\frac{\eta^{c*}-1}{\eta^{c*}}}\right]^{\frac{\eta^{c*}}{\eta^{c*}-1}} \qquad (87)$$

subject to:

$$p^{c*} c^* \geq \sum_j p_j^{c*} c_j^* \qquad (88)$$

where: η^{c*} is the elasticity of substitution between different consumption goods, θ_j^{c*} is a weighting parameter with $\sum_j \theta_j^{c*} = 1$, c^* is the aggregate consumption level, p^{c*} is the aggregate consumption price, and p_j^{c*}, c_j^* are the price and quantity of good j.

The first order condition for optimality implies:

$$c_j^* = \theta_j^{c*} \left(\frac{p_j^{c*}}{p^{c*}}\right)^{-\eta^{c*}} c^* \qquad (89)$$

where the price of aggregate consumption p^{c*} is a function of individual goods prices and their weights in the consumption bundle:

$$p^{c*} = \left[\sum_j \theta_j^{c*} \left(p_j^{c*} \right)^{1-\eta^{c*}} \right]^{\frac{1}{1-\eta^{c*}}} \quad (90)$$

Once a decision is made at the top level, the representative consumer will move to the second level, which involves the choice between varieties of the same goods that are produced in different countries, subject to the value of aggregate expenditure decided at the previous level.

At the second level, the representative consumer has the following maximisation problem:

$$\max_{c_{j,k}^*} c_j^* = \left[\sum_k \theta_{j,k}^{c*\frac{1}{\eta_j^{c*}}} c_{j,k}^{*\frac{\eta_j^{c*}-1}{\eta_j^{c*}}} \right]^{\frac{\eta_j^{c*}}{\eta_j^{c*}-1}} \quad (91)$$

subject to:

$$p_j^{c*} c_j^* \geq \sum_k p_{j,k}^y c_{j,k}^* \quad (92)$$

where η_j^{c*} is the elasticity of substitution between varieties of the same good that are produced in different countries, $\theta_{j,k}^{c*}$ is a weighting parameter with $\sum_k \theta_{j,k}^{c*} = 1$, and $p_{j,k}^y, c_{j,k}^*$ are the price and quantity of variety exported from country k.

The first order condition for optimality in turn implies:

$$c_{j,k}^* = \theta_{j,k}^{c*} \left(\frac{p_{j,k}^y}{p_j^{c*}} \right)^{-\eta_j^{c*}} c_j^* \quad (93)$$

where the price of the consumption good p_j^{c*} is a function of the prices of different varieties and their weights in the variety bundle:

$$p_j^{c*} = \left[\sum_k \theta_{j,k}^{c*} p_{j,k}^{y\ 1-\eta_j^{c*}} \right]^{\frac{1}{1-\eta_j^{c*}}} \quad (94)$$

As in Imbs and Méjean (2010), we can combine equations (89) and (93) to have:

$$c_{j,k}^* = \theta_{j,k}^{c*} \left(\frac{p_{j,k}^y}{p_j^{c*}} \right)^{-\eta_j^{c*}} \theta_j^{c*} \left(\frac{p_j^{c*}}{p^{c*}} \right)^{-\eta^{c*}} c^* \quad (95)$$

From this we can derive the demand for Australian exports as in equation (23) of the paper.

Appendix E: Distribution sector's optimisation problem

As discussed in Section 2 of the paper, a notional domestic distribution sector combines imported goods and domestically produced goods to form composite consumption $c_t^{a,\ell}$, investment $i_{j,t}$, intermediate goods $z_{j,t}$, and government spending g_t.

Consumption of goods and services

For each household, the distribution sector first chooses the consumption bundle that maximises the value of composite consumption:

$$\max_{\{c_{j,t}^{a,\ell}\}} c_t^{a,\ell} = \left(\sum_j \theta_j^c \left(c_{j,t}^{a,\ell} \right)^{\frac{\eta^c - 1}{\eta^c}} \right)^{\frac{\eta^c}{\eta^c - 1}}$$

subject to the allocated budget:

$$p_t^c c_t^{a,\ell} \geq \sum_j p_{j,t}^c c_{j,t}^{a,\ell}$$

Following Obstfeld and Rogoff (1996, Chapter 4), we can obtain the optimal demand for each good:

$$c_{j,t}^{a,\ell} = \theta_j^c c_t^{a,\ell} \left(\frac{p_{j,t}^c}{p_t^c} \right)^{-\eta^c}$$

and the price index for the composite consumption good:

$$p_t^c = \left(\sum_j \theta_j^c \left(p_{j,t}^c \right)^{1-\eta^c} \right)^{\frac{1}{1-\eta^c}}$$

The distribution sector then chooses from a basket of goods composed of domestically produced $cd_{j,t}^{a,\ell}$ and imported $cm_{j,t}^{a,\ell}$ varieties, to maximise the value of consumption:

$$\max_{cd_{j,t}^{a,\ell}, cm_{j,t}^{a,\ell}} c_{j,t}^{a,\ell} = \left(\left(1 - \theta_j^{cm}\right) \left(cd_{j,t}^{a,\ell} \right)^{\frac{\eta_j^{cm} - 1}{\eta_j^{cm}}} + \left(\theta_j^{cm} \right) \left(cm_{j,t}^{a,\ell} \right)^{\frac{\eta_j^{cm} - 1}{\eta_j^{cm}}} \right)^{\frac{\eta_j^{cm}}{\eta_j^{cm} - 1}}$$

subject to the allocated expenditure:

$$p_{j,t}^{c} c_{j,t}^{a,\ell} \geq (1+\omega_{j,t}^{c}\tau_{t}^{gst})(1+\tau_{j,t}^{oit,c}) p_{j,t}^{y} cd_{j,t}^{a,\ell} + (1+\omega_{j,t}^{c}\tau_{t}^{gst})(1+\tau_{j,t}^{oit,c}) p_{j,t}^{m} cm_{j,t}^{a,\ell}$$

The optimal demand for domestic and imported goods is again as follows:

$$cd_{j,t}^{a,\ell} = (1-\theta_{j}^{cm}) c_{j,t}^{a,\ell} \left(\frac{\left(1+\omega_{j,t}^{c}\tau_{t}^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^{d}}{p_{j,t}^{c}} \right)^{-\eta_{j}^{cm}}$$

$$cm_{j,t}^{a,\ell} = \theta_{j}^{cm} c_{j,t}^{a,\ell} \left(\frac{\left(1+\omega_{j,t}^{c}\tau_{t}^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^{m}}{p_{j,t}^{c}} \right)^{-\eta_{j}^{cm}}$$

And the price index for composite consumption good j is:

$$p_{j,t}^{c} = \left(\begin{array}{l} (1-\theta_{j}^{cm})\left(\left(1+\omega_{j,t}^{c}\tau_{t}^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^{y}\right)^{1-\eta_{j}^{cm}} \\ +\theta_{j}^{cm}\left(\left(1+\omega_{j,t}^{c}\tau_{t}^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^{m}\right)^{1-\eta_{j}^{cm}} \end{array} \right)^{\frac{1}{1-\eta_{j}^{cm}}}$$

Investment goods and services

For capital formation of sector j, the distribution sector first chooses the investment goods bundle to maximise the value of the firm's gross fixed capital expenditure:

$$\max_{\{i_{j,k,t}\}} i_{j,t} = \left(\sum_{k} \theta_{j,k}^{i} \left(i_{j,k,t} \right)^{\frac{\eta_{j}^{i}-1}{\eta_{j}^{i}}} \right)^{\frac{\eta_{j}^{i}}{\eta_{j}^{i}-1}}$$

subject to the budget constraint:

$$p_{j,t}^{i} i_{j,t} \geq \sum_{k} p_{j,k,t}^{i} i_{j,k,t}$$

The optimal demand and implied price index functions are as follows:

$$i_{j,k,t} = \theta_{j,k}^{i} i_{j,t} \left(\frac{p_{j,k,t}^{i}}{p_{j,t}^{i}} \right)^{-\eta_{j}^{i}}$$

$$p_{j,t}^{i} = \left(\sum_{k} \theta_{j,k}^{i} \left(p_{j,k,t}^{i} \right)^{1-\eta_{j}^{i}} \right)^{\frac{1}{1-\eta_{j}^{i}}}$$

The distribution sector then chooses from a variety of domestic and imported investment goods to maximise the value of investment:

$$\max_{id_{j,k,t}, im_{j,k,t}} i_{j,k,t} = \left(\left(1-\theta^{im}_{j,k}\right)\left(id_{j,k,t}\right)^{\frac{\eta^{im}_{j,k}-1}{\eta^{im}_{j,k}}} + \left(\theta^{im}_{j,k}\right)\left(im_{j,k,t}\right)^{\frac{\eta^{im}_{j,k}-1}{\eta^{im}_{j,k}}} \right)^{\frac{\eta^{im}_{j,k}}{\eta^{im}_{j,k}-1}}$$

subject to the allocated budget:

$$p^{i}_{j,k,t} i_{j,k,t} \geq (1+\omega^{i}_{j,k,t}\tau^{gst}_{t})(1+\tau^{oit,i}_{j,k,t}) p^{y}_{j,t} id_{j,k,t} + (1+\omega^{i}_{j,k,t}\tau^{gst}_{t})(1+\tau^{oit,i}_{j,k,t}) p^{m}_{j,t} im_{j,k,t}$$

The optimal demand for domestic and imported goods is as follows:

$$id_{j,k,t} = \left(1-\theta^{im}_{j,k}\right) i_{j,k,t} \left(\frac{\left(1+\omega^{i}_{j,t,k}\tau^{gst}_{t}\right)\left(1+\tau^{oit,i}_{j,t,k}\right) p^{y}_{k,t}}{p^{i}_{j,k,t}} \right)^{-\eta^{im}_{j,k}}$$

$$im_{j,k,t} = \theta^{im}_{j,k} i_{j,k,t} \left(\frac{\left(1+\omega^{i}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,i}_{j,k,t}\right) p^{m}_{k,t}}{p^{i}_{j,k,t}} \right)^{-\eta^{im}_{j,k}}$$

$$p^{i}_{j,k,t} = \left(\begin{array}{l} \left(1-\theta^{im}_{j,k}\right)\left(\left(1+\omega^{i}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,i}_{j,k,t}\right) p^{y}_{k,t}\right)^{1-\eta^{im}_{j,k}} \\ +\theta^{im}_{j,k}\left(\left(1+\omega^{i}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,i}_{j,k,t}\right) p^{m}_{k,t}\right)^{1-\eta^{im}_{j,k}} \end{array} \right)^{\frac{1}{1-\eta^{im}_{j,k}}}$$

Intermediate goods and services

Similarly, for intermediate input of sector j, the distribution sector first chooses the intermediate goods bundle to maximise the value of the firm's expenditure:

$$\max_{\{z_{j,k,t}\}} z_{j,t} = \left(\sum_{k} \theta^{z}_{j,k} \left(z_{j,k,t}\right)^{\frac{\eta^{z}_{j}-1}{\eta^{z}_{j}}} \right)^{\frac{\eta^{z}_{j}}{\eta^{z}_{j}-1}}$$

subject to the budget constraint:

$$p^{z}_{j,t} z_{j,t} \geq \sum_{k} p^{z}_{j,k,t} z_{j,k,t}$$

The optimal demand and implied price index functions are as follows:

$$z_{j,k,t} = \theta^{z}_{j,k} z_{j,t} \left(\frac{p^{z}_{j,k,t}}{p^{z}_{j,t}} \right)^{-\eta^{z}_{i}}$$

$$p^{z}_{jt} = \left(\sum_{k} \theta^{z}_{j,k} \left(p^{z}_{j,k,t} \right)^{1-\eta^{z}_{j}} \right)^{\frac{1}{-\eta^{z}_{j}}}$$

The distribution sector then chooses from a variety of domestic and imported intermediate goods to maximise the value of the firm's use of intermediate goods:

$$\max_{zd_{j,k,t}, zm_{j,k,t}} z_{j,k,t} = \left(\left(1-\theta^{zm}_{j,k}\right) \left(zd_{j,k,t} \right)^{\frac{\eta^{zm}_{j,k}-1}{\eta^{zm}_{j,k}}} + \left(\theta^{zm}_{j,k}\right) \left(zm_{j,k,t} \right)^{\frac{\eta^{zm}_{j,k}-1}{\eta^{zm}_{j,k}}} \right)^{\frac{\eta^{zm}_{j,k}}{\eta^{zm}_{j,k}-1}}$$

subject to the allocated budget:

$$p^{z}_{j,k,t} z_{j,k,t} \geq (1+\omega^{z}_{j,k,t} \tau^{gst}_{t})(1+\tau^{oit,z}_{j,k,t}) p^{y}_{j,t} zd_{j,k,t} + (1+\omega^{z}_{j,k,t} \tau^{gst}_{t})(1+\tau^{oit,z}_{j,k,t}) p^{m}_{j,t} zm_{j,k,t}$$

The optimal demand and implied price index functions are as follows:

$$zd_{j,k,t} = \left(1-\theta^{zm}_{j,k}\right) z_{j,k,t} \left(\frac{\left(1+\omega^{z}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,z}_{j,k,t}\right) p^{y}_{k,t}}{p^{z}_{j,k,t}} \right)^{-\eta^{zm}_{j,k}}$$

$$zm_{j,k,t} = \theta^{zm}_{j,k} z_{j,k,t} \left(\frac{\left(1+\omega^{z}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,z}_{j,k,t}\right) p^{m}_{k,t}}{p^{z}_{j,k,t}} \right)^{-\eta^{zm}_{j,k}}$$

$$p^{z}_{j,k,t} = \left(\begin{array}{l} \left(1-\theta^{zm}_{j,k}\right)\left(\left(1+\omega^{z}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,z}_{j,k,t}\right) p^{y}_{k,t}\right)^{1-\eta^{zm}_{j,k}} \\ +\theta^{zm}_{j,k}\left(\left(1+\omega^{z}_{j,k,t}\tau^{gst}_{t}\right)\left(1+\tau^{oit,z}_{j,k,t}\right) p^{m}_{k,t}\right)^{1-\eta^{zm}_{j,k}} \end{array} \right)^{\frac{1}{1-\eta^{zm}_{j,k}}}$$

Government purchases of goods and services

As discussed in the paper, the government is assumed to consume a fixed bundle of goods and services $g_{j,t}$ supplied by the distribution sector. Given the market price of each good $p^g_{j,t}$, total government spending is:

$$p^g_t g_t = \sum_j p^g_{j,t} g_{j,t}$$

and aggregate real government expenditure is:

$$g_t = \left(\sum_j \theta^g_j \left(g_{j,t} \right)^{\frac{\eta^g - 1}{\eta^g}} \right)^{\frac{\eta^g}{\eta^g - 1}}$$

The distribution sector then chooses from a variety of domestic and imported goods to maximise the value of the government's purchase of goods:

$$\max_{gd_{j,t}, gm_{jt}} g_{j,t} = \left(\left(1 - \theta^{gm}_j\right) \left(gd_{j,t} \right)^{\frac{\eta^{gm}_j - 1}{\eta^{gm}_j}} + \left(\theta^{gm}_j \right) \left(gm_{j,t} \right)^{\frac{\eta^{gm}_j - 1}{\eta^{gm}_j}} \right)^{\frac{\eta^{gm}_j}{\eta^{gm}_j - 1}}$$

subject to the budget constraint:

$$p^g_{j,t} g_{j,t} \geq p^y_{j,t} gd_{j,t} + p^m_{j,t} gm_{j,t}$$

The optimal demand and implied price index functions are as follows:

$$gd_{j,t} = (1 - \theta^{gm}_j) g_{j,t} \left(\frac{p^y_{j,t}}{p^g_{j,t}} \right)^{-\eta^{gm}_j}$$

$$gm_{j,t} = \theta^{gm}_j g_{j,t} \left(\frac{p^m_{j,t}}{p^g_{j,t}} \right)^{-\eta^{gm}_j}$$

$$p^g_{j,t} = \left((1 - \theta^{gm}_j) \left(p^y_{j,t} \right)^{1 - \eta^{gm}_j} + \theta^{gm}_j \left(p^m_{j,t} \right)^{1 - \eta^{gm}_j} \right)^{\frac{1}{1 - \eta^{gm}_j}}$$

Calibration of CES weights

The optimal demand conditions we have derived above for the distribution sector suggest the following relationship between the CES weights and other variables:

$$\theta_j^c = \frac{c_{j,t}^{a,\ell} p_{j,t}^c}{c_t^{a,\ell} p_t^c} \left(\frac{p_{j,t}^c}{p_t^c} \right)^{\eta^c - 1} \tag{96}$$

$$\theta_{j,k}^i = \frac{i_{j,k,t} p_{j,k,t}^i}{i_{j,t} p_{j,t}^i} \left(\frac{p_{j,k,t}^i}{p_{j,t}^i} \right)^{\eta_j^i - 1} \tag{97}$$

$$\theta_{j,k}^z = \frac{z_{j,k,t} p_{j,k,t}^z}{z_{j,t} p_{j,t}^Z} \left(\frac{p_{j,k,t}^z}{p_{j,t}^z} \right)^{\eta_j^z - 1} \tag{98}$$

$$\theta_j^g = \frac{g_{j,t} p_{j,t}^g}{g_t p_t^g} \left(\frac{p_{j,t}^g}{p_t^g} \right)^{\eta^g - 1} \tag{99}$$

Here $\frac{c_{j,t}^{a,\ell} p_{j,t}^c}{c_t^{a,\ell} p_t^c}$, $\frac{i_{j,k,t} p_{j,k,t}^i}{i_{j,t} p_{j,t}^i}$, $\frac{z_{j,k,t} p_{j,k,t}^z}{z_{j,t} p_{j,t}^z}$ and $\frac{g_{j,t} p_{j,t}^g}{g_t p_t^g}$ are the cost share of each good in consumption, government spending, investment, and intermediate use implied by Table 3 of the paper. We also have:

$$\theta_j^{cm} = \frac{cm_{j,t}^{a,\ell} \left(1+\omega_{j,t}^c \tau_t^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^m}{c_{j,t}^{a,\ell} p_{j,t}^c} \left(\frac{\left(1+\omega_{j,t}^c \tau_t^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right) p_{j,t}^m}{p_{j,t}^c} \right)^{\eta_j^{cm} - 1} \tag{100}$$

$$\theta_{j,k}^{im} = \frac{im_{j,k,t} \left(1+\omega_{j,k,t}^i \tau_t^{gst}\right)\left(1+\tau_{j,k,t}^{oit,i}\right) p_{k,t}^m}{i_{j,k,t} p_{j,k,t}^i} \left(\frac{\left(1+\omega_{j,k,t}^i \tau_t^{gst}\right)\left(1+\tau_{j,k,t}^{oit,i}\right) p_{k,t}^m}{p_{j,k,t}^i} \right)^{\eta_{j,k}^{im} - 1} \tag{101}$$

$$\theta_{j,k}^{zm} = \frac{zm_{j,k,t} \left(1+\omega_{j,k,t}^z \tau_t^{gst}\right)\left(1+\tau_{j,k,t}^{oit,z}\right) p_{k,t}^m}{z_{j,k,t} p_{j,k,t}^z} \left(\frac{\left(1+\omega_{j,k,t}^z \tau_t^{gst}\right)\left(1+\tau_{j,k,t}^{oit,z}\right) p_{k,t}^m}{p_{j,k,t}^z} \right)^{\eta_{j,k}^{zm} - 1} \tag{102}$$

$$\theta_j^{gm} = \frac{gm_{j,t} p_{j,t}^m}{g_{j,t} p_{j,t}^g} \left(\frac{p_{j,t}^m}{p_{j,t}^g} \right)^{\eta_j^{gm} - 1} \tag{103}$$

Here

$$\frac{cm_{j,t}^{a,\ell}\left(1+\omega_{j,t}^{c}\tau_{t}^{gst}\right)\left(1+\tau_{j,t}^{oit,c}\right)p_{j,t}^{m}}{c_{j,t}^{a,\ell}\,p_{j,t}^{c}},$$

$$\frac{im_{j,k,t}\left(1+\omega_{j,k,t}^{i}\tau_{t}^{gst}\right)\left(1+\tau_{j,k,t}^{oit,i}\right)p_{k,t}^{m}}{i_{j,k,t}\,p_{j,k,t}^{i}},$$

$$\frac{zm_{j,k,t}\left(1+\omega_{j,k,t}^{z}\tau_{t}^{gst}\right)\left(1+\tau_{j,k,t}^{oit,z}\right)p_{k,t}^{m}}{z_{j,k,t}\,p_{j,k,t}^{z}} \text{ and}$$

$$\frac{gm_{j,t}\,p_{j,t}^{m}}{g_{j,t}\,p_{j,t}^{g}}$$

are the cost shares of imports in consumption, government spending, investment, and intermediate use as implied by Table 3 and

Table 8 of the paper.

As these relations hold for all periods including the base year of the model, we can then use equations (86) to (93) and the cost share information from Table 3 and

Table 8 to estimate the CES weights $\theta_{j}^{c},\theta_{j,k}^{i},\theta_{j,k}^{z},\theta_{j}^{g},\theta_{j}^{cm},\theta_{j,k}^{im},\theta_{j,k}^{zm},\theta_{j}^{gm}$.

Appendix F: Gauss-Seidel algorithm for solving OLGA

Following Auerbach and Kotlikoff (1987), we solve the Gauss-Seidel algorithm to solve the model. Specifically, the computation of OLGA's equilibrium responses to a policy scenario has three stages:

(i) Solving for the long-run steady state of the economy before the assumed change in policy occurs;

(ii) Solving for the long-run steady state which the economy eventually converges to after the policy takes effect; and

(iii) Solving for the transition path that the economy takes between these two steady states.

Each stage involves the following six steps:

Step 1: Start with an initial estimate for the average wage level and government policies, and each sector's labour input or output level and goods prices.

Step 2: Solve the firm's problem for each sector given labour input (or output level) and goods prices. This determines each sector's wage, output and demand for capital, labour and intermediate goods.

Step 3: Solve each household's problem given the average wage and goods prices. This determines labour supplied, and demand for consumption goods.

Step 4: Solve export and government demand for each sector given goods prices, and calculate the government's total tax revenue and expenditure.

Step 5: Check if all goods markets clear, if wage is equalised across all sectors, if aggregate labour inputs equals labour supplied, and if the government's budget constraint holds.

Step 6: If Step 5 is not passed, update the estimate using the quasi-Newton method as suggested in Ludwig (2007) and Heer and Maussner (2009); and return to Step 2.

Appendix G: Sectoral concordance between OLGA and ABS Input-Output Industry Group (IOIG)

IOIG	Description	OLGA Sector	Code
0101	Sheep, Grains, Beef and Dairy Cattle	Agriculture	AGR
0102	Poultry and Other Livestock	Agriculture	AGR
0103	Other Agriculture	Agriculture	AGR
0201	Aquaculture	Agriculture	AGR
0301	Forestry and Logging	Agriculture	AGR
0401	Fishing, Hunting and Trapping	Agriculture	AGR
0501	Agriculture, Forestry and Fishing Support Services	Agriculture	AGR
0601	Coal Mining	Mining	MIN
0701	Oil and Gas Extraction	Mining	MIN
0801	Iron Ore Mining	Mining	MIN
0802	Non Ferrous Metal Ore Mining	Mining	MIN
0901	Non Metallic Mineral Mining	Mining	MIN
1001	Exploration and Mining Support Services	Mining	MIN
1101	Meat and Meat product Manufacturing	Manufacturing	MAN
1102	Processed Seafood Manufacturing	Manufacturing	MAN
1103	Dairy Product Manufacturing	Manufacturing	MAN
1104	Fruit and Vegetable Product Manufacturing	Manufacturing	MAN
1105	Oils and Fats Manufacturing	Manufacturing	MAN
1106	Grain Mill and Cereal Product Manufacturing	Manufacturing	MAN
1107	Bakery Product Manufacturing	Manufacturing	MAN
1108	Sugar and Confectionery Manufacturing	Manufacturing	MAN
1109	Other Food Product Manufacturing	Manufacturing	MAN
1201	Soft Drinks, Cordials and Syrup Manufacturing	Manufacturing	MAN
1202	Beer Manufacturing	Manufacturing	MAN
1205	Wine, Spirits and Tobacco	Manufacturing	MAN
1301	Textile Manufacturing	Manufacturing	MAN
1302	Tanned Leather, Dressed Fur and Leather Product Manufacturing	Manufacturing	MAN
1303	Textile Product Manufacturing	Manufacturing	MAN
1304	Knitted Product Manufacturing	Manufacturing	MAN

1305	Clothing Manufacturing	Manufacturing	MAN
1306	Footwear Manufacturing	Manufacturing	MAN
1401	Sawmill Product Manufacturing	Manufacturing	MAN
1402	Other Wood Product Manufacturing	Manufacturing	MAN
1501	Pulp, Paper and Paperboard Manufacturing	Manufacturing	MAN
1502	Paper Stationery and Other Converted Paper Product Manufacturing	Manufacturing	MAN
1601	Printing (including the reproduction of recorded media)	Manufacturing	MAN
1701	Petroleum and Coal Product Manufacturing	Manufacturing	MAN
1801	Human Pharmaceutical and Medicinal Product Manufacturing	Manufacturing	MAN
1802	Veterinary Pharmaceutical and Medicinal Product Manufacturing	Manufacturing	MAN
1803	Basic Chemical Manufacturing	Manufacturing	MAN
1804	Cleaning Compounds and Toiletry Preparation Manufacturing	Manufacturing	MAN
1901	Polymer Product Manufacturing	Manufacturing	MAN
1902	Natural Rubber Product Manufacturing	Manufacturing	MAN
2001	Glass and Glass Product Manufacturing	Manufacturing	MAN
2002	Ceramic Product Manufacturing	Manufacturing	MAN
2003	Cement, Lime and Ready-Mixed Concrete Manufacturing	Manufacturing	MAN
2004	Plaster and Concrete Product Manufacturing	Manufacturing	MAN
2005	Other Non-Metallic Mineral Product Manufacturing	Manufacturing	MAN
2101	Iron and Steel Manufacturing	Manufacturing	MAN
2102	Basic Non-Ferrous Metal Manufacturing	Manufacturing	MAN
2201	Forged Iron and Steel Product Manufacturing	Manufacturing	MAN
2202	Structural Metal Product Manufacturing	Manufacturing	MAN
2203	Metal Containers and Other Sheet Metal Product manufacturing	Manufacturing	MAN
2204	Other Fabricated Metal Product manufacturing	Manufacturing	MAN
2301	Motor Vehicles and Parts; Other Transport Equipment manufacturing	Manufacturing	MAN
2302	Ships and Boat Manufacturing	Manufacturing	MAN
2303	Railway Rolling Stock Manufacturing	Manufacturing	MAN
2304	Aircraft Manufacturing	Manufacturing	MAN
2401	Professional, Scientific, Computer and Electronic Equipment Manufacturing	Manufacturing	MAN
2403	Electrical Equipment Manufacturing	Manufacturing	MAN
2404	Domestic Appliance Manufacturing	Manufacturing	MAN
2405	Specialised and other Machinery and Equipment Manufacturing	Manufacturing	MAN
2501	Furniture Manufacturing	Manufacturing	MAN

2502	Other Manufactured Products	Manufacturing	MAN
2601	Electricity Generation	Utilities	UTL
2605	Electricity Transmission, Distribution, On Selling and Electricity Market Operation	Utilities	UTL
2701	Gas Supply	Utilities	UTL
2801	Water Supply, Sewerage and Drainage Services	Utilities	UTL
2901	Waste Collection, Treatment and Disposal Services	Utilities	UTL
3001	Residential Building Construction	Construction	CST
3002	Non-Residential Building Construction	Construction	CST
3101	Heavy and Civil Engineering Construction	Construction	CST
3201	Construction Services	Construction	CST
3301	Wholesale Trade	Services	SRV
3901	Retail Trade	Services	SRV
4401	Accommodation	Services	SRV
4501	Food and Beverage Services	Services	SRV
4601	Road Transport	Services	SRV
4701	Rail Transport	Services	SRV
4801	Water, Pipeline and Other Transport	Services	SRV
4901	Air and Space Transport	Services	SRV
5101	Postal and Courier Pick-up and Delivery Service	Services	SRV
5201	Transport Support services and storage	Services	SRV
5401	Publishing (except Internet and Music Publishing)	Services	SRV
5501	Motion Picture and Sound Recording	Services	SRV
5601	Broadcasting (except Internet)	Services	SRV
5701	Internet Service Providers, Internet Publishing and Broadcasting, Websearch Portals and Data Processing	Services	SRV
5801	Telecommunication Services	Services	SRV
6001	Library and Other Information Services	Services	SRV
6201	Finance	Services	SRV
6301	Insurance and Superannuation Funds	Services	SRV
6401	Auxiliary Finance and Insurance Services	Services	SRV
6601	Rental and Hiring Services (except Real Estate)	Services	SRV
6701	Ownership of Dwellings	Dwellings	DWE
6702	Non-Residential Property Operators and Real Estate Services	Services	SRV
6901	Professional, Scientific and Technical Services	Services	SRV

7001	Computer Systems Design and Related Services	Services	SRV
7210	Employment, Travel Agency and Other Administrative Services	Services	SRV
7310	Building Cleaning, Pest Control and Other Support Services	Services	SRV
7501	Public Administration and Regulatory Services	Services	SRV
7601	Defence	Services	SRV
7701	Public Order and Safety	Services	SRV
8010	Primary and Secondary Education Services (incl Pre-Schools and Special Schools)	Services	SRV
8110	Technical, Vocational and Tertiary Education Services (incl undergraduate and postgraduate)	Services	SRV
8210	Arts, Sports, Adult and Other Education Services (incl community education)	Services	SRV
8401	Health Care Services	Services	SRV
8601	Residential Care and Social Assistance Services	Services	SRV
8901	Heritage, Creative and Performing Arts	Services	SRV
9101	Sports and Recreation	Services	SRV
9201	Gambling	Services	SRV
9401	Automotive Repair and Maintenance	Services	SRV
9402	Other Repair and Maintenance	Services	SRV
9501	Personal Services	Services	SRV
9502	Other Services	Services	SRV